ELECTRICAL FAULTS

1 Introduction

The power system, namely the electrical network, must satisfy the demand at all times without exceeding the limits imposed on bus voltages and transmitted powers in order to ensure normal operation. In addition, the system must be protected against faults that may arise at any moment without warning. A fault is a short circuit between the phases of the network.

When a fault appears in the network, the circuit breakers operate and isolate the fault. These circuit breakers are rated for a fault current (electric arc extinction) and for the voltage at which they must open the faulted line.

The fault level, or short-circuit power, at a point in the network is defined as the product of the voltage just before the fault appears and the fault current. In fault studies, the fault currents and voltages must be determined.

2 Analysis of electrical faults

The selection of protection equipment is based on the magnitude of fault currents. Therefore, all types of faults must be studied beforehand. Different cases may arise (Fig. 1).

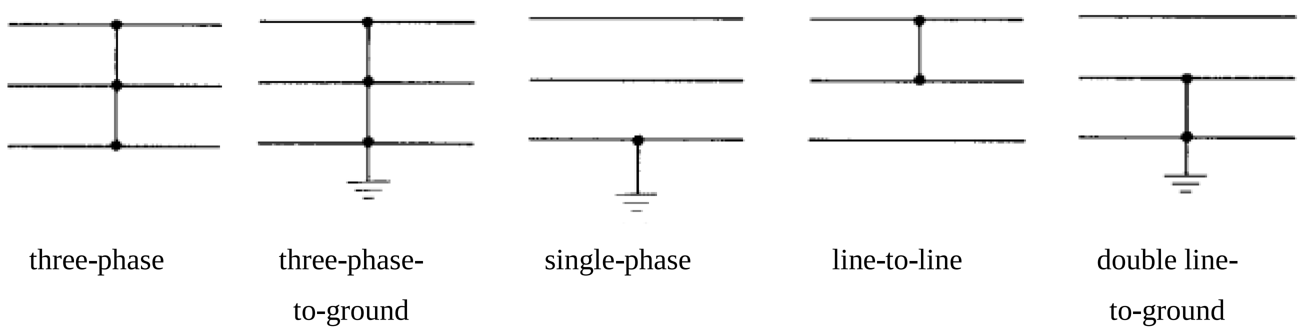


Fig. 1 : Types of faults

2.1 Characteristics of short-circuit faults

- Durations : self-extinguishing, transient, or permanent.

- Origins:

▪ mechanical (broken conductors, accidental electrical connection between two conductors by a foreign conductive body such as tools or animals);

▪ electrical overvoltages of internal or atmospheric origin;

▪ insulation degradation due to heat, humidity, or a corrosive environment.

- Locations: internal or external to a machine or a switchboard.

- Types:
  - Single-phase: 80% of cases;
  - Two-phase: 15% of cases. These faults often develop into three-phase faults;
  - Three-phase: only 5% from the outset.

The most frequent fault is the phase-to-ground fault, whereas the most severe are the three-phase and three-phase-to-ground faults. The equipment affected, in order, is overhead lines, cables, transformers, and then switchgear. Lightning is the most frequent cause.

2.2 Consequences of short-circuit faults

They vary according to the nature and duration of the faults, the point in the installation concerned, and the current magnitude:

- At the fault point, the presence of fault arcs, with:
  - deterioration of insulation,
  - melting of conductors,
  - fire and danger to people;

- For the faulty circuit:
  - electrodynamic forces, with deformation of busbars and tearing of cables;
  - overheating due to increased Joule losses, with a risk of insulation deterioration;

- For the other electrical circuits of the affected network or nearby networks:
  - voltage dips during the fault-clearing time, from a few milliseconds to a few hundred milliseconds;
  - outage of a greater or smaller part of the network depending on its configuration and the selectivity of its protections;
  - dynamic instability and/or loss of synchronism of machines;
  - disturbances in control circuits;
  - etc.

2.3 Fault analysis

- Determination of the maximum and minimum values of three-phase (symmetrical) fault currents;

- Determination of unsymmetrical fault currents (single phase-to-ground, line-to-line, double line-to-ground, ...);

- Rating of circuit breakers and fuses: breaking and making capacities of devices;

- Capability of electrical equipment: electrodynamic withstand of conductors and switchgear, thermal withstand of cables under overcurrents;

- Selection of protection types and setting of protection relays;

- Determination of voltage levels during a fault;

- Specification of transformer and generator impedances.

The magnitudes of fault currents depend on:

- network impedances: line impedances, connections, transformer impedances, connections, and grounding resistances;
- internal impedances of generators;
- fault resistance (arc resistance).

Note: Generator impedances

The generator behavior is divided into three periods: subtransient period (first cycles), transient period (covering a relatively longer time), and steady-state period.
Subtransient period, with $X_G = X''_d$: used to determine the breaking capacity of HV circuit breakers and the operating time of the protection relay system for high-voltage networks.
Transient period, $X_G = X'_d$: used to determine the breaking capacity of MV circuit breakers and the operating time of the protection relay system for medium-voltage networks.

3 Calculation of symmetrical faults

3.1 Thévenin method

The most common approach is to replace the network and its generators by a Thévenin circuit seen from the fault terminals. This circuit is represented by a voltage source having the value just before the fault appears (representing the voltage at the node where the fault occurs) in series with an equivalent Thévenin impedance representing the network seen from the faulted node (Fig. 2).

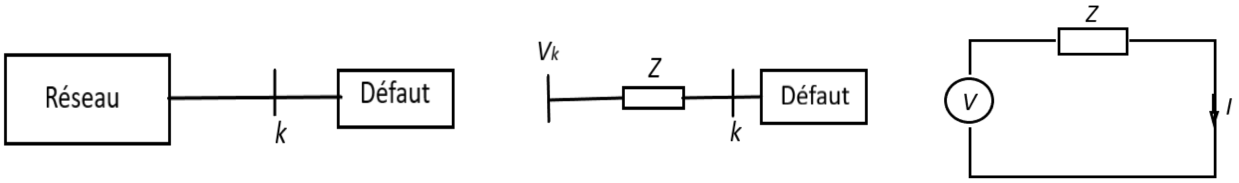


Fig. 2 : Thévenin equivalent diagram

$$Z_{pu} = \frac{Z}{V/I_n} = \frac{I_n Z}{V} \quad \rightarrow \quad Z = \frac{V.Z_{pu}}{I_n} \tag{1}$$

with I_n the rated current (full load).
The fault or short-circuit current is:

$$I_f = \frac{V}{Z} = \frac{I_n}{Z_{pu}} \tag{2}$$

The short-circuit power is then:

$$S_{cc} = \sqrt{3} V I_f = \frac{\sqrt{3} V I_n}{Z_{pu}} = \frac{S_b}{Z_{pu}} \quad (3)$$

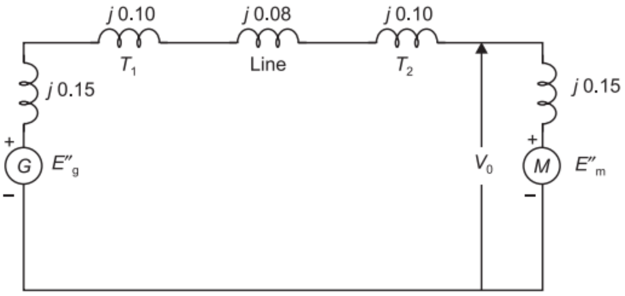
The short-circuit power S_{cc} measures the electrical strength of the node, expressed in MVA, and thus makes it possible to size the busbars and the breaking capacity of the circuit breakers.

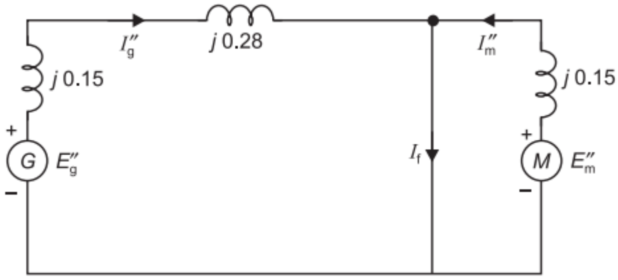
Example 1:

Consider the system in the figure below. The data are:
G1: 20 MVA; 12.66 kV; $X'' = 15\ \%$ (subtransient reactance)
M: 20 MVA; 12.66 kV; $X'' = 15\ \%$ (subtransient reactance)
T1: 20 MVA; 12.66/66 kV; $X = 10\ \%$
T2: 20 MVA; 66/12.66 kV; $X = 10\ \%$
The motor consumes 10 MW with a leading power factor of 0.8 at a voltage of 11 kV when a three-phase fault occurs at the motor terminals. Determine the generator current, the motor current, and the fault current.

Solution :

$S_b = 20\ MVA$





Pre-fault equivalent diagram

Equivalent diagram during the fault

Pre-fault condition:

$$V_0 = \frac{11}{12{,}66} \angle 0° = 0{,}8688 \angle 0°\ pu \qquad P_L = \frac{10}{20} = 0{,}50\ pu$$

$$I_0 = \frac{0{,}50}{0{,}8688 \times 0{,}80} = 0{,}7194 \angle 36{,}87°\ pu$$

The $f.e.m$ of the generator and $f.c.e.m$ of the motor (subtransient regime):

$$E_g'' = V_0 + j(0{,}15 + 0{,}10 + 0{,}08 + 0{,}10) I_0 = 0{,}8688 + j0{,}43 \times 0{,}7194 \angle 36{,}87° = 0{,}7266 \angle 19{,}9°\ pu$$

$$E''_m = V_0 - j0{,}15 I_0 = 0{,}8688 - j0{,}15 \times 0{,}7194\angle 36{,}87° = 0{,}9374\angle -5{,}28°\ pu$$

The currents:

$$I''_g = \frac{E''_g}{j(0{,}15 + 0{,}28)} = \frac{0{,}72\ 66\angle 19{,}9°}{0{,}43\angle 90°} = 1{,}689\angle -70{,}1° = (0{,}575 - j1{,}588)\ pu$$

$$I''_m = \frac{E''_m}{j0{,}15} = \frac{0{,}9374\angle -5{,}28°}{0{,}15\angle 90°} = 6{,}25\angle -95{,}28° = (-0{,}575 - j6{,}233)\ pu$$

The fault current:

$$I_f = I''_g + I''_m = 0{,}575 - j1{,}588 - 0{,}575 - j6{,}233 = -j7{,}811\ pu$$

In actual values :

$$I_b = \frac{20 \times 1000}{\sqrt{3} \times 12{,}66} = 912{,}085\ A$$

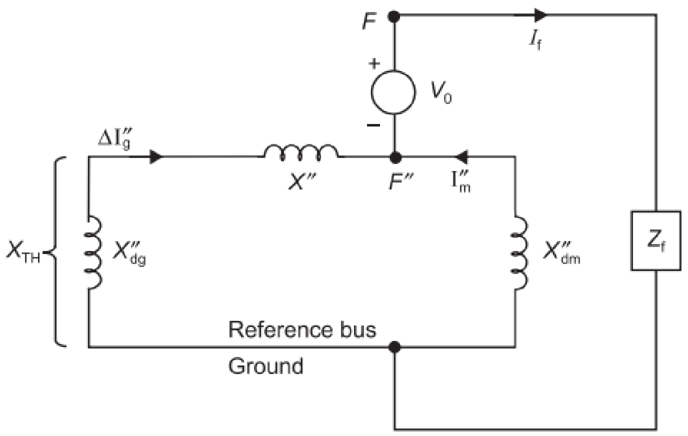
$$I''_g = 912{,}085 \times 1{,}689\angle -70{,}1° = 1540{,}5\angle -70{,}1°\ A$$

$$I''_m = 912{,}085 \times 6{,}25\angle -95{,}28° = 5700{,}5\angle -95{,}28°\ A$$

$$I_f = 912{,}085 \times (-j7{,}811) = 7124{,}3\angle -90°\ A$$

Thévenin method:

The equivalent Thévenin diagram during the fault is given by the following figure.



With: $X'' = j(0{,}1 + 0{,}08 + 0{,}1) = j0{,}28$; $X''_{dg} = j0{,}15$; $X''_{dm} = j0{,}15\ pu$

Therefore: $X''_{dg} + X'' = j(0{,}15 + 0{,}28) = j0{,}43\ pu$

$$X_{Th} = \frac{(X''_{dg} + X'')(X''_{dm})}{X''_{dg} + X'' + X''_{dm}} = \frac{j0{,}43 \times j0{,}15}{j(0{,}43 \times 0{,}15)} = j0{,}1112\ pu$$

$$I_f = \frac{V_0}{Z_f + X_{Th}} = \frac{0{,}8688\angle 0°}{j0{,}1112} = -j7{,}811\ pu$$

Change in generator current:

$$\Delta I_g'' = I_f \times \frac{X_{dm}''}{X_{dg}'' + X'' + X_{dm}''} = -j7{,}811 \times \frac{j0{,}15}{j(0{,}15 + 0{,}28 + 0{,}15)} = -j2{,}02\, pu$$

Likewise:

$$\Delta I_m'' = I_f \times \frac{X_{dg}'' + X''}{X_{dg}'' + X'' + X_{dm}''} = -j7{,}811 \times \frac{j(0{,}15 + 0{,}28)}{j(0{,}15 + 0{,}28 + 0{,}15)} = -j5{,}79\, pu$$

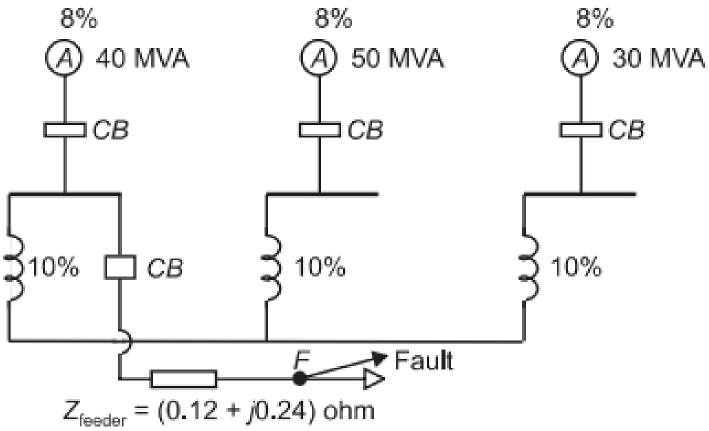
Therefore:

$$I_g'' = \Delta I_g'' + I_0 = -j2{,}02 + 0{,}7194\angle 36{,}87° = (0{,}575 - j1{,}589) pu$$

$$I_m'' = \Delta I_m'' - I_0 = -j5{,}79 - 0{,}7194\angle 36{,}87° = (-0{,}575 - j6{,}221) pu$$

Example 2:

Three 11.2 kV generators are connected by a tie busbar through current-limiting reactors (figure below). A feeder is supplied from generator A busbar at 11.2 kV. The feeder impedance is (0.12+j0.24) Ω per phase. Calculate the maximum power (MVA) that can be supplied for a symmetrical short circuit at the remote end of the feeder.



Solution :

Choice of base: $S_b = 50\, MVA$; $U_b = 11{,}2\, kV$

Reactance calculations:

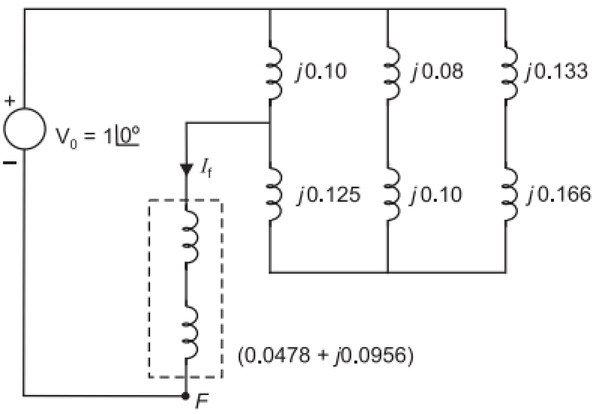
$$x_{Ag} = j0{,}08 \times \frac{50}{40} = j0{,}10\, pu \qquad x_{Bg} = j0{,}08 \times \frac{50}{50} = j0{,}08\, pu \qquad x_{Cg} = j0{,}08 \times \frac{50}{30} = j0{,}133\, pu$$

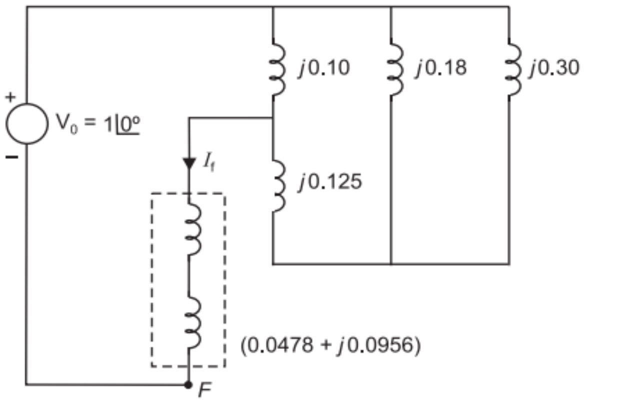
$$x_A = j0{,}10 \times \frac{50}{40} = j0{,}125\, pu \qquad x_B = j0{,}10 \times \frac{50}{50} = j0{,}10\, pu \qquad x_C = j0{,}10 \times \frac{50}{30} = j0{,}166\, pu$$

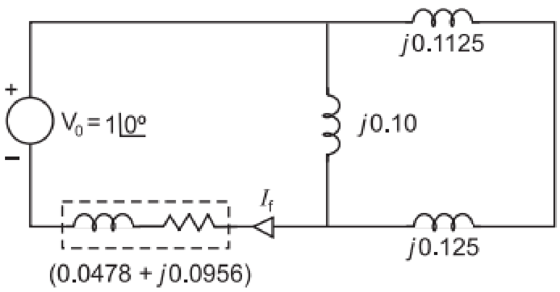
$$Z_b = \frac{(U_b)^2}{S_b} = \frac{(11{,}2)^2}{50} = 2{,}5088\, \Omega$$

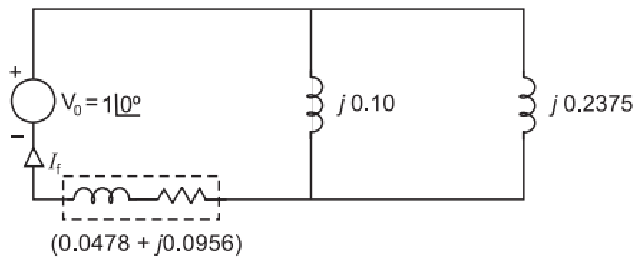
$$Z_{feeder} = \frac{0{,}12 + j0{,}24}{2{,}5088} = (0{,}0478 + j0{,}0956)\, pu$$

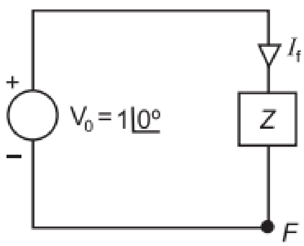
Assume a zero pre-fault current (i.e. no load before the fault). The equivalent diagram during the fault and its simplification are given below.











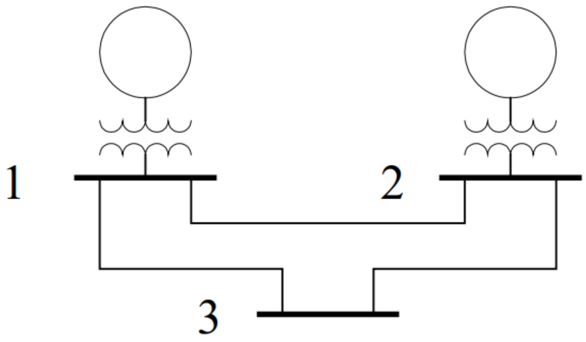
$$Z = 0,0478 + j0,0956 + j\frac{0,10 \times 0,2375}{0,3375}$$
$$= 0,1727\angle 73,94°\ pu$$

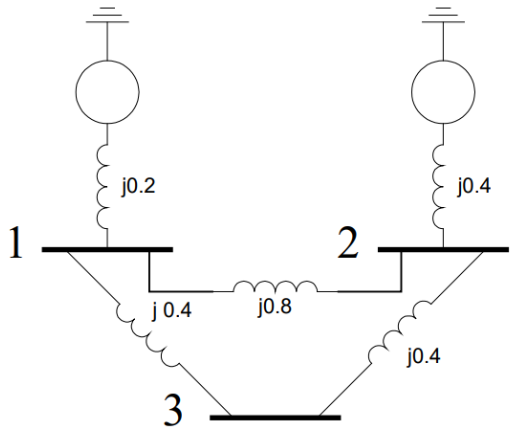
The short-circuit power :

$$S_{cc} = (V_0 \cdot I_f) \times S_b = \left(V_0 \cdot \frac{V_0}{Z}\right) \times S_b = \frac{V_0^2}{Z} \times S_b = \frac{(1)^2}{0,1727} \times 50 = 289,5\ MVA$$

Example 3:

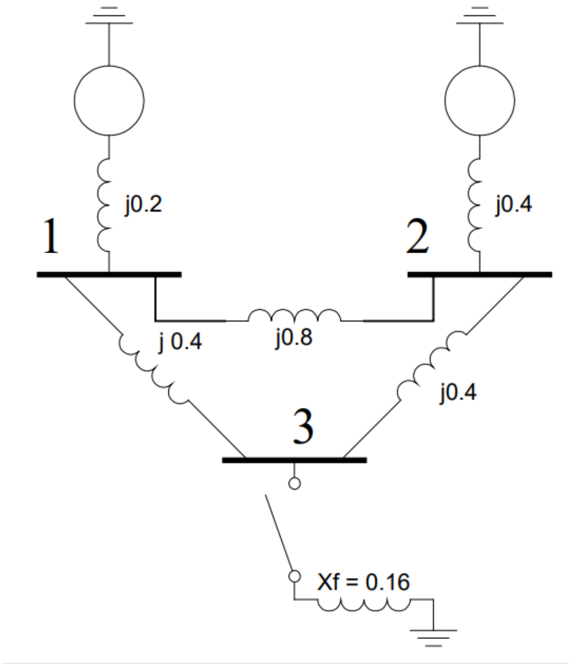
Consider the 3-node network in the figure below. A three-phase fault occurs at node 3 through an impedance $Z_f = j0,16\ pu$. Calculate the fault current and the short-circuit power.

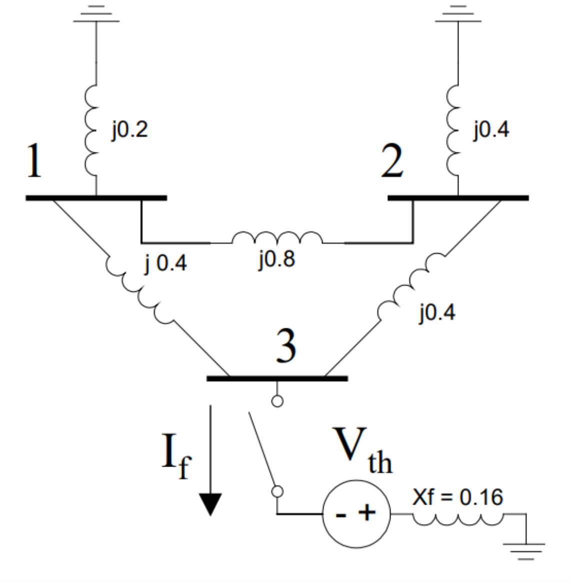




Solution:

The equivalent Thévenin circuit and the simplification steps are shown below.

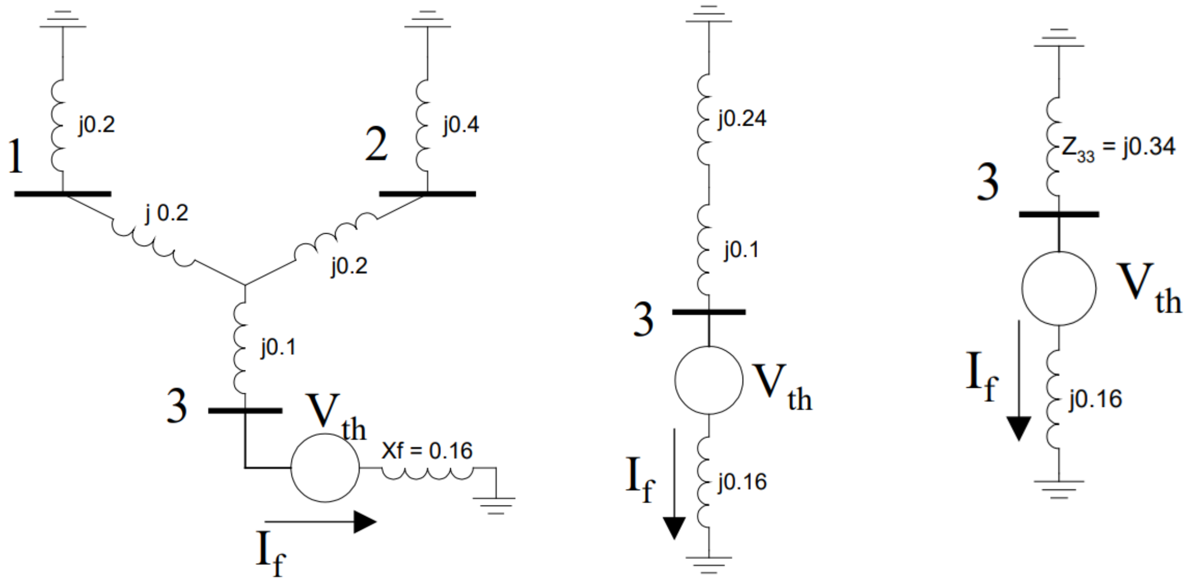




Applying the delta-to-star transformation:

$$Z_{10} = Z_{20} = \frac{(j0{,}4)(j0{,}8)}{(j1{,}6)} = j0{,}2\ pu$$

$$Z_{30} = \frac{(j0{,}4)(j0{,}4)}{(j1{,}6)} = j0{,}1\ pu$$



The equivalent Thévenin impedance:

$$Z_{33} = \frac{(j0{,}4)(j0{,}6)}{j0{,}4 + j0{,}6} + j0{,}1 = j0{,}24 + j0{,}1$$

$$= j0{,}34\ pu$$

$$V_1^0 = V_2^0 = V_3^0 = 1{,}0\ pu$$

Therefore:

$$I_3^f = \frac{V_3^0}{Z_{33} + Z_f} = \frac{1{,}0}{j0{,}34 + j0{,}16} = -j2{,}0\ pu$$

$$S_b = 100\ MVA$$

$$S_{cc3} = \frac{S_b}{Z_{33}} = \frac{100}{0{,}34} = 294\ MVA$$

3.2 Impedance-matrix method

Reducing the network by the Thévenin method is not efficient and is difficult to apply to large networks. The matrix approach is therefore used.

Each generator is represented by a $f.e.m$ constant source behind the appropriate reactance, transient or subtransient (depending on the circuit breaker and its speed: X' if it is slow and X'' if it is fast $X' > X''$). The lines are represented by their equivalent π model. To preserve linearity, the loads are converted into a constant-impedance model.

The pre-fault nodal voltages can be obtained from the results of a power-flow calculation.

The fault is simulated by connecting a fault impedance to the faulted node.

Consider a network with n nodes. The first step is to obtain the pre-fault nodal voltages and line currents using a power-flow calculation. The resulting pre-fault nodal voltage vector is denoted by:

$$V^0 = \begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_k^0 \\ \vdots \\ V_n^0 \end{bmatrix} \tag{4}$$

Let node k be the location of a balanced three-phase fault and Z_f the impedance of this fault (Fig. 3).

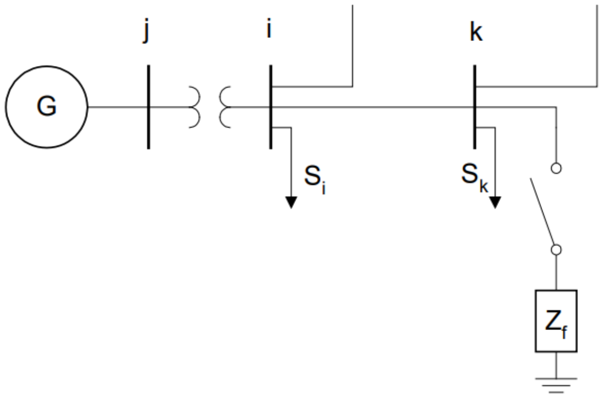


Fig. 3: Network with n nodes with a symmetrical fault

The network change caused by the fault is equivalent to placing a fault voltage at the faulted node while short-circuiting all other sources. Fig. 4 shows the equivalent Thévenin network with the generators replaced by the transient/subtransient reactances and their $f.e.m$ short-circuited.

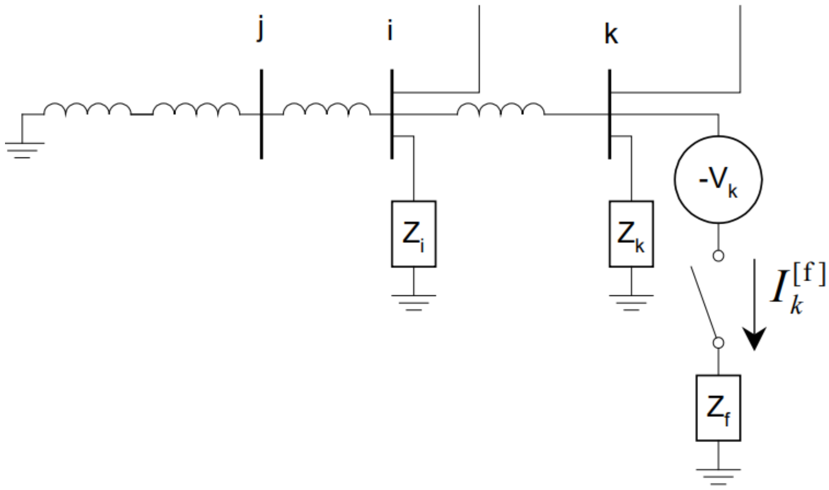


Fig. 4: Equivalent network with the fault

The post-fault voltage vector (after the fault occurs) is given by:

$$V^f = V^0 + \Delta V \tag{5}$$

where ΔV is the vector of voltage changes caused by the fault.

$$I = Y_{bus} V \tag{6}$$

$$I^f = Y_{bus} \Delta V \tag{7}$$

I^f is the vector of injected currents: the network is injected only by the current $-I_f$ at node k. It is therefore written as :

$$I^f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_k^f = -I_f \\ \vdots \\ 0 \end{bmatrix} \tag{8}$$

From (7) :

$$\Delta V = Z_{bus} I^f \tag{9}$$

Where $Z_{bus} = Y_{bus}^{-1}$ is the impedance matrix of the Thévenin network.

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -I_k^f \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Therefore:

$$\Delta V_k = -Z_{kk} I_f \tag{11}$$

The voltage at node k under fault is

$$V_k^f = V_k^0 + \Delta V_k = V_k^0 - Z_{kk} I_f \tag{12}$$

and also

$$V_k^f = Z_f I_f \tag{13}$$

From equations (12) and (13) :

$$Z_f I_f = V_k^0 - Z_{kk} I_f \tag{14}$$

$$I_f = \frac{V_k^0}{Z_{kk}+Z_f} \tag{15}$$

At node i according to equation (10):

$$\Delta V_i = -Z_{ik} I_f \tag{16}$$

Then

$$V_i^f = V_i^0 - Z_{ik} I_f \tag{17}$$

From equations (15) and (17), we obtain:

$$V_i^f = V_i^0 - \frac{Z_{ik}}{Z_{kk}+Z_f} V_k^0 \tag{18}$$

For $i = k$, equation (18) becomes:

$$V_k^f = V_k^0 - \frac{Z_{kk}}{Z_{kk}+Z_f} V_k^0 = \frac{Z_f}{Z_{kk}+Z_f} V_k^0 \tag{19}$$

Note that synchronous motors must be included in the formulation of Z_{bus} whereas load impedances can be neglected because they are very high compared with generator and transmission-line impedances. Nevertheless, if load impedances are taken into account, they are calculated by replacing the load power using the pre-fault voltage:

$$Z_{i,load} = \frac{\left|V_i^0\right|^2}{S_{i,load}^*} \tag{20}$$

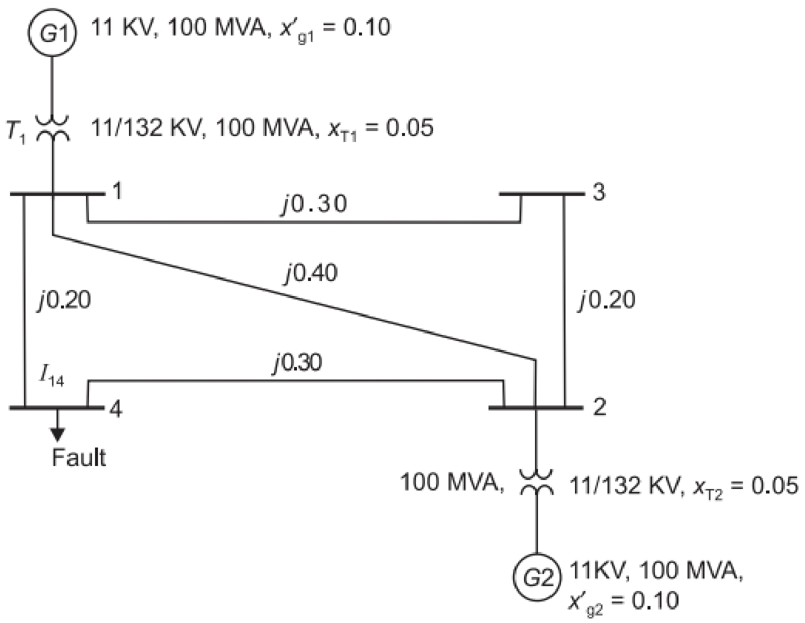
The fault current flowing from node i to node j is given by:

$$I_{ij}^f = y_{ij}(V_i^f - V_j^f) \tag{21}$$

The generator current i after the fault is given by:

$$I_{Gi}^f = \frac{V'_{Gi} - V_i^f}{jX'_{Gi}} \tag{22}$$

Example 4 :



Solution:

Assume pre-fault nodal voltages equal to 1 pu and zero pre-fault currents.

Formation of the admittance matrix and obtaining the impedance matrix:

$$Y_{11} = \frac{1}{j0{,}30} + \frac{1}{j0{,}30} + \frac{1}{j0{,}2} + \frac{1}{j0{,}4} = -j14{,}166$$

$Y_{12} = Y_{21} = \frac{-1}{j0{,}4} = j2{,}5$ $\quad$ $Y_{13} = Y_{31} = \frac{-1}{j0{,}3} = j3{,}333$ $\quad$ $Y_{14} = Y_{41} = j5{,}0$

$Y_{22} = -j14{,}166$ $\quad$ $Y_{23} = Y_{32} = j5{,}0$ $\quad$ $Y_{24} = Y_{42} = j3{,}333$

$Y_{33} = -j8{,}333$ $\quad$ $Y_{34} = Y_{43} = 0{,}0$ $\quad$ $Y_{44} = -j8{,}333$

$$Y_{bus} = \begin{bmatrix} -j14{,}166 & j2{,}5 & j3{,}333 & j5{,}0 \\ j2{,}5 & -j14{,}166 & j5{,}0 & j3{,}333 \\ j3{,}333 & j5{,}0 & -j8{,}333 & 0{,}0 \\ j5{,}0 & j3{,}333 & 0{,}0 & -j8{,}333 \end{bmatrix}$$

$$Z_{bus} = {Y_{bus}}^{-1} = \begin{bmatrix} j0{,}1806 & j0{,}1194 & j0{,}1438 & j0{,}1560 \\ j0{,}1194 & j0{,}1806 & j0{,}1560 & j0{,}1438 \\ j0{,}1438 & j0{,}1560 & j0{,}2712 & j0{,}1486 \\ j0{,}1560 & j0{,}1438 & j0{,}1486 & j0{,}2712 \end{bmatrix}$$

Using equation (18) :

$$V_1^0 = V_2^0 = V_3^0 = V_4^0 = 1{,}0\ pu$$

$$k = 4 \quad \text{and} \quad Z_f = 0{,}0$$

$$V_1^f = V_1^0 - \frac{Z_{14}}{Z_{44}} V_4^0 = 1{,}0 - \frac{j0{,}1560}{j0{,}2712} 1{,}0 = 0{,}4247\ pu$$

$$V_2^f = V_2^0 - \frac{Z_{24}}{Z_{44}} V_4^0 = 1{,}0 - \frac{j0{,}1438}{j0{,}2712} 1{,}0 = 0{,}4697\ pu$$

$$V_3^f = V_3^0 - \frac{Z_{34}}{Z_{44}} V_4^0 = 1{,}0 - \frac{j0{,}1486}{j0{,}2712} 1{,}0 = 0{,}4520\ pu$$

$$V_4^f = 0{,}0$$

The fault currents are calculated using equation (21):

$$I_{12}^f = y_{12}\left(V_1^f - V_2^f\right) = \frac{(0{,}4247 - 0{,}4697)}{j0{,}4} = j0{,}1125\ pu$$

$$I_{13}^f = y_{13}\left(V_1^f - V_3^f\right) = \frac{(0{,}4247 - 0{,}4520)}{j0{,}3} = j0{,}091\ pu$$

$$I_{14}^f = y_{14}\left(V_1^f - V_4^f\right) = \frac{(0{,}4247 - 0{,}0)}{j0{,}2} = -j2{,}1235\ pu$$

$$I_{24}^f = y_{24}\left(V_2^f - V_4^f\right) = \frac{(0{,}4697 - 0{,}0)}{j0{,}3} = -j1{,}5656\ pu$$

$$I_{23}^f = y_{23}\left(V_2^f - V_3^f\right) = \frac{(0{,}4697 - 0{,}452)}{j0{,}2} = -j0{,}0885\ pu$$

Example 5:

Repeat Example 3 using the impedance-matrix method.

Solution :

$$Y_{bus} = \begin{bmatrix} -j8{,}75 & j1{,}25 & j2{,}50 \\ j1{,}25 & -j6{,}25 & j2{,}50 \\ j2{,}50 & j2{,}50 & -j5{,}00 \end{bmatrix}$$

$$Y_{bus} = Y_{bus}{}^{-1} = \begin{bmatrix} j0{,}16 & j0{,}08 & j0{,}12 \\ j0{,}08 & j0{,}24 & j0{,}16 \\ j0{,}12 & j0{,}16 & j0{,}34 \end{bmatrix}$$

$$I_3^f = \frac{V_3^0}{Z_{33}+Z_f} = \frac{1{,}0}{j0{,}34 + j0{,}16} = -j2{,}0\,pu$$

$$V_1^f = V_1^0 - Z_{13} I_3^f = 1{,}0 - (j0{,}12)(-j2{,}0) = 0{,}76\,pu$$

$$V_2^f = V_2^0 - Z_{23} I_3^f = 1{,}0 - (j0{,}16)(-j2{,}0) = 0{,}68\,pu$$

$$V_3^f = V_3^0 - Z_{33} I_3^f = 1{,}0 - (j0{,}34)(-j2{,}0) = 0{,}32\,pu$$

4 Symmetrical components

An unbalanced linear three-phase system can be transformed into a set of 3 symmetrical circuits: positive-sequence balanced system (denoted d or 1), negative-sequence balanced system (denoted i or 2), and zero-sequence system (denoted h or 0). Thus, an abc system of voltages or currents can be represented by the superposition of the positive-, negative-, and zero-sequence systems as follows:

$$\begin{aligned} F_a &= F_{a0} + F_{ad} + F_{ai} \\ F_b &= F_{b0} + F_{bd} + Fb_i \\ F_a &= F_{c0} + F_{cd} + F_{ci} \end{aligned} \tag{23}$$

zero-sequence system: $$F_{a0} = F_{b0} = F_{c0} = F_0 \tag{24}$$

positive-sequence system: $$F_{ad} = F_d \quad F_{bd} = a^2 F_{ad} = a^2 F_d \quad F_{cd} = aF_{ad} = aF_d \tag{25}$$

negative-sequence system: $$F_{ai} = F_i \quad F_{bi} = aF_{ai} = aF_i \quad F_{ci} = a^2 F_{ai} = a^2 F_i \tag{26}$$

Let:

$$\begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} F_0 \\ F_d \\ F_i \end{bmatrix} \tag{27}$$

and conversely

$$\begin{bmatrix} F_0 \\ F_d \\ F_i \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} \tag{28}$$

With

$$a = e^{j\frac{2\pi}{3}} = 1\angle 120° \tag{29}$$

We verify: $a^3 = 1$; $a^2 + a + 1 = 0$

We define the transformation matrix:

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{30}$$

And its inverse :

$$T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = \frac{1}{3}T^* \tag{31}$$

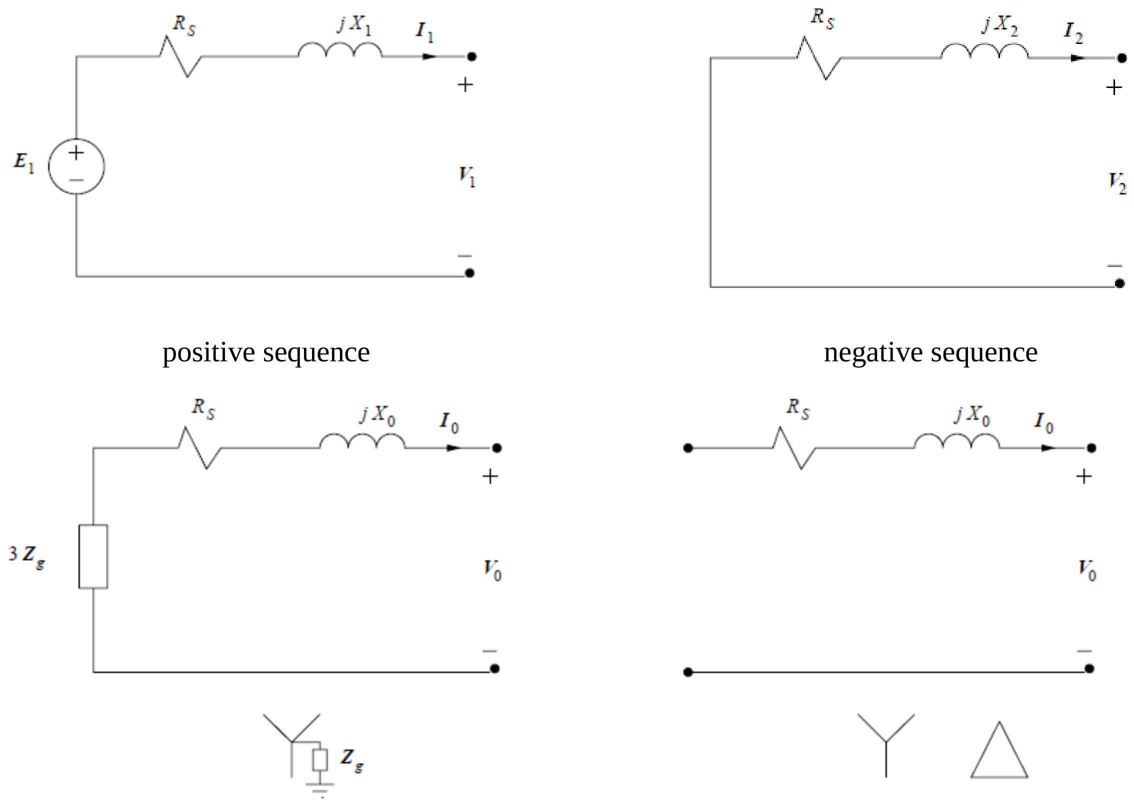
In compact form:

$$F_{abc} = T \cdot F_{odi} \tag{32}$$

$$F_{odi} = T^{-1} \cdot F_{abc} \tag{33}$$

Based on this transformation, the various network elements can be represented by models associated with the symmetrical sequences separately.

4.1 Generators



zero sequence

Fig. 5: symmetrical sequences of a synchronous generator

X_d generally correspond to the direct-axis transient and subtransient reactances $X_d{}'$ and $X_d{}''$ respectively, depending on the reaction time of the circuit breakers ($X_d{}' > X_d{}''$). The negative-sequence reactance is approximately equal to:

$$X_i = \frac{X_d + X_q}{2} \tag{34}$$

X_q is the corresponding quadrature-axis reactance. For a smooth-rotor machine: $X_i = X_d$.
The value of the zero-sequence reactance is often such that $X_0 << X_d$.
It should be noted that only the positive sequence presents a source voltage because the machine generates only the positive sequence in steady state.

4.2 Transformers

In positive and negative sequence, the transformer is modeled in the usual way.

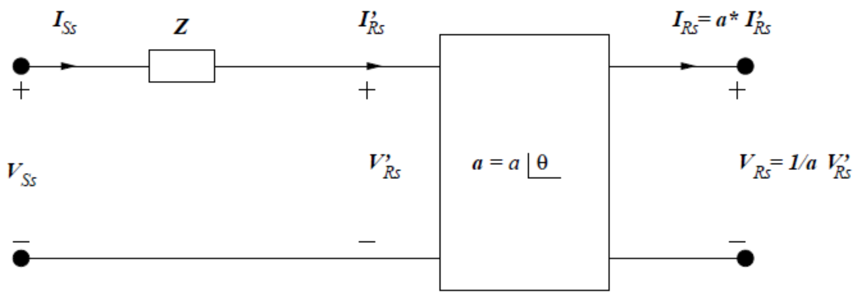


Fig. 6: Positive- and negative-sequence transformer networks

The phase shift is given according to the case by the following table:

connection	θ	
	positive sequence	negative sequence
Y-Y	0°	0°
Δ-Δ	0°	0°
Δ-Y	-30°	+30°
Y-Δ	+30°	-30°

If $\theta = 0$ (actual transformation ratio), then the classical model in π :

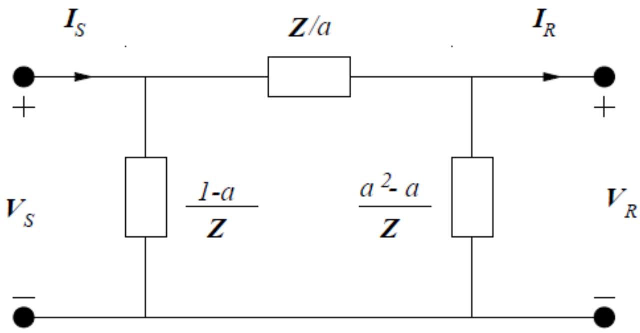


Fig. 7: π equivalent circuit of the transformer

For the zero sequence, the model depends on the connection type (Fig. 8).

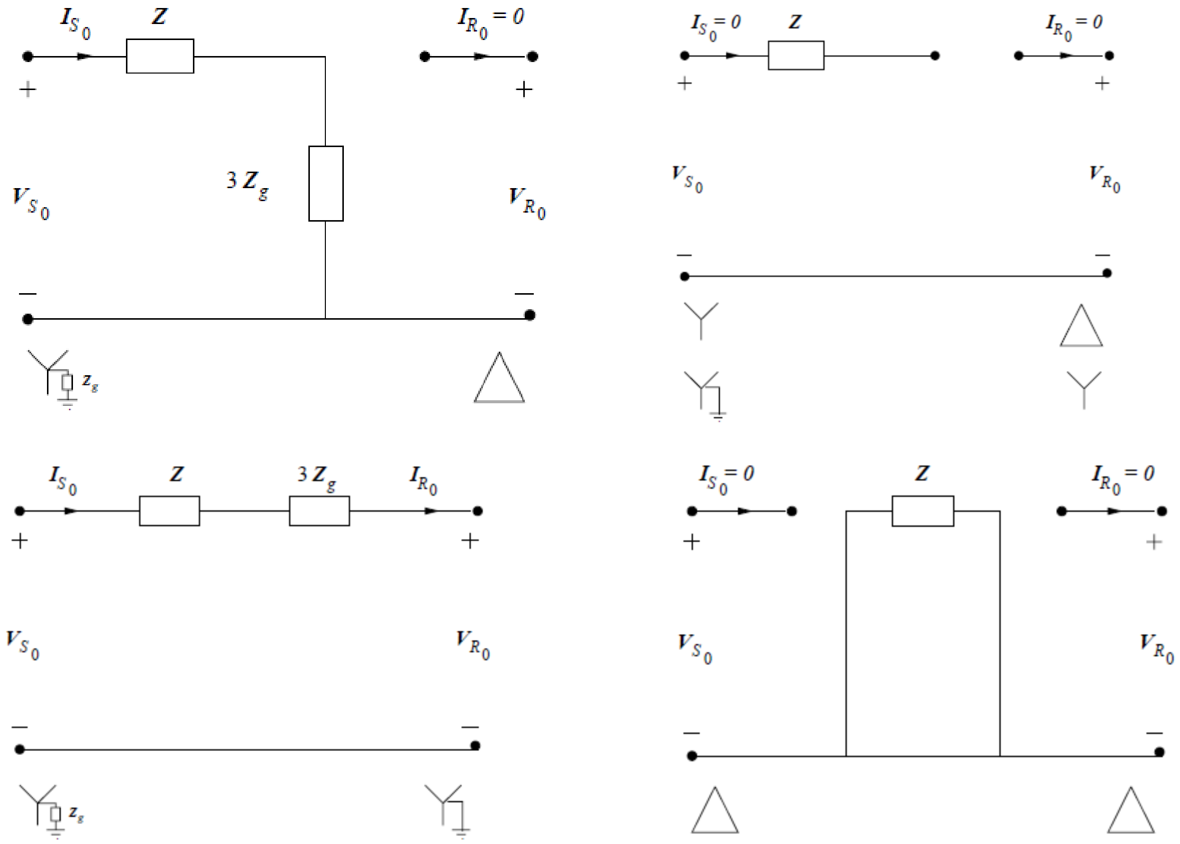


Fig. 8: zero-sequence network according to the transformer connection type

4.3 Transmission lines

The lines are modeled by the equivalent π for each sequence with:

$$Z_d = Z_i \quad \text{and} \quad Y_d = Y_i \tag{35}$$

$$Z_0 = 3 \div 4\, Z_d \quad \text{and} \quad Y_d = 3 \div 4\, Y_0 \tag{36}$$

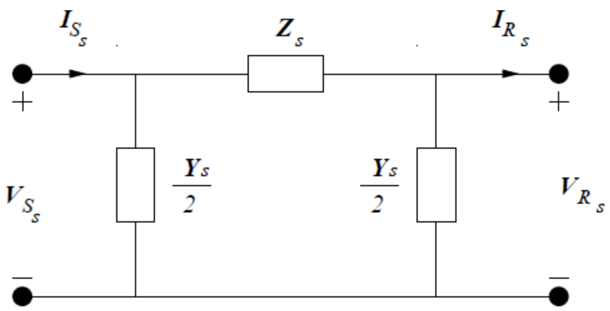
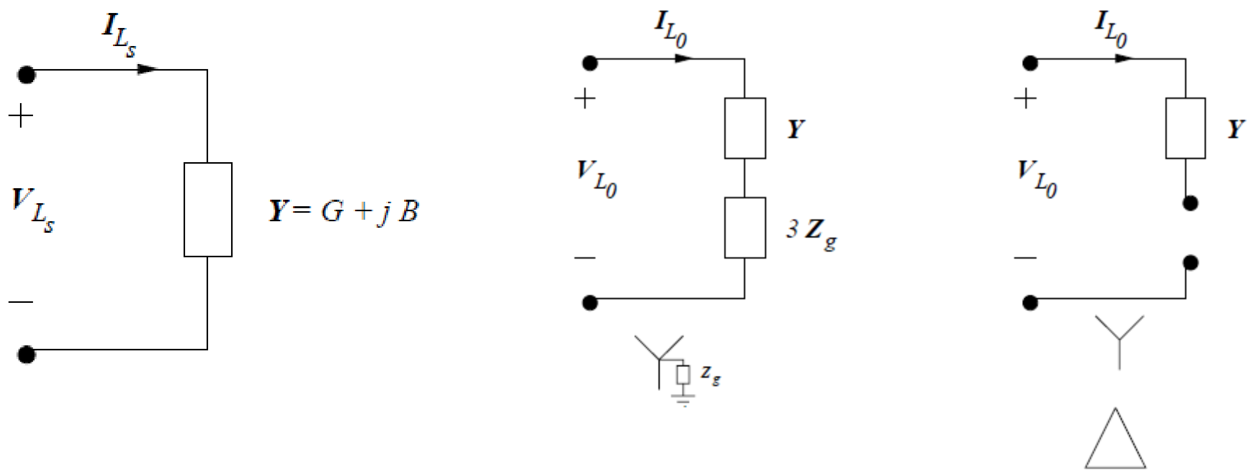


Fig. 9: π model of lines for the different sequences

4.4 Loads



positive / negative sequence zero sequence

Fig. 10 : sequence representation for loads

We have :

$$G = \frac{P_L}{V_L^2} \tag{37}$$

$$B = Q_L / V_L^2 \tag{38}$$

$V_L = V_L^0$: voltage just before the fault, or pre-fault voltage

$P_L,\ Q_L$: active and reactive load powers

Star-connected load impedance

A balanced three-phase load with a self-impedance Z_p and mutual impedance Z_m is illustrated in Fig. 11; the load neutral is grounded through an impedance Z_n.

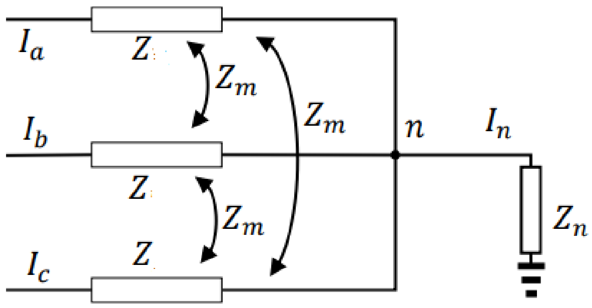


Fig. 11: Balanced load connected in star

The phase voltages are :

$$\begin{cases} V_a = Z_p I_a + Z_m I_b + Z_m I_c + Z_n I_n \\ V_b = Z_m I_a + Z_p I_b + Z_m I_c + Z_n I_n \\ V_c = Z_m I_a + Z_m I_b + Z_p I_c + Z_n I_n \end{cases} \quad (39)$$

The current in the neutral is:

$$I_n = I_a + I_b + I_c \quad (40)$$

Replacing I_n in the system of equations, we obtain the matrix form:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_p + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_p + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_p + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (41)$$

In compact form:

$$V_{abc} = Z_{abc} . I_{abc} \quad (42)$$

Applying the symmetrical-components transformation, we obtain

$$T \cdot V_{odi} = Z_{abc} \cdot T \cdot I_{odi} \quad (43)$$

$$V_{odi} = T^{-1} \cdot Z_{abc} \cdot T \cdot I_{odi} \quad (44)$$

Or

$$V_{odi} = Z_{0di} \cdot I_{odi} \quad (45)$$

With

$$Z_{odi} = T^{-1} Z_{abc} T = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_d & 0 \\ 0 & 0 & Z_i \end{bmatrix} \quad (46)$$

$$Z_d = Z_i = Z_p - Z_m \quad (47)$$

$$Z_0 = Z_p + 2Z_m + 3Z_n \quad (48)$$

5 Unsymmetrical faults (unbalanced)

All equipment being represented by impedances, for each sequence the system can be represented by its Thévenin equivalent: $f.e.m\ E$ (pre-fault voltage at the considered node) in series with an impedance.

5.1 Single phase-to-ground fault

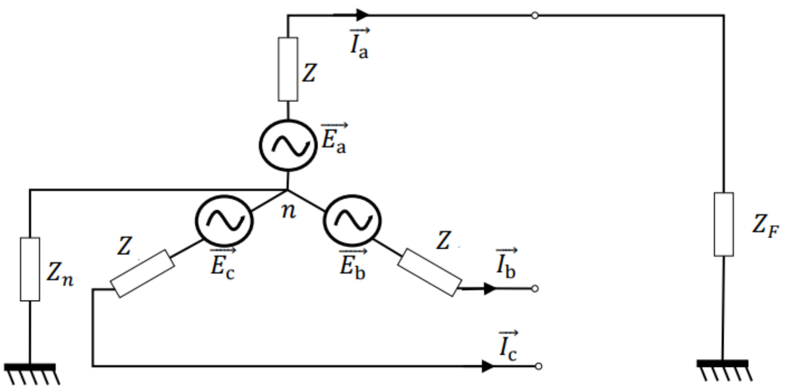


Fig. 12: Fault on phase a of an unloaded generator

A balanced three-phase synchronous generator without load, with its neutral grounded through an impedance Z_n is shown in Fig. 12. Suppose that a phase-to-ground fault, called a zero-sequence fault, occurs on phase a through an impedance Z_F.

Since the generator is unloaded, the following conditions exist at the fault point:

$$I_b = I_c = 0 \tag{49}$$

$$V_a = Z_F . I_a \tag{50}$$

The symmetrical components of the currents can be calculated by:

$$\begin{bmatrix} I_0 \\ I_d \\ I_i \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \tag{51}$$

The values of the symmetrical components of the fault current I_a are therefore:

$$I_d = I_i = I_0 = \frac{1}{3} I_a \tag{52}$$

If phase a is taken as the reference: $E_a = E\ \ (\rightarrow E_b = a^2 E)$

The voltage of phase a can be expressed in terms of symmetrical components:

$$V_a = V_d + V_i + V_0\ =\ (E - Z_d . I_d) - Z_i . I_i - Z_0 . I_0 \tag{53}$$

By substituting the currents into equation (53):

$$V_a = E - (Z_d + Z_i + Z_0).I_0 \tag{54}$$

We also have :

$$V_a = Z_F.I_a = 3.Z_F.I_0 \tag{55}$$

Therefore:

$$E - (Z_d + Z_i + Z_0).I_0 = 3.Z_F.I_0 \tag{56}$$

Then:

$$I_0 = \frac{E}{Z_d + Z_i + Z_0 + 3Z_F} \tag{57}$$

The fault current is therefore:

$$I_f = I_a = 3I_0 = \frac{3E}{Z_d + Z_i + Z_0 + 3Z_F} \tag{58}$$

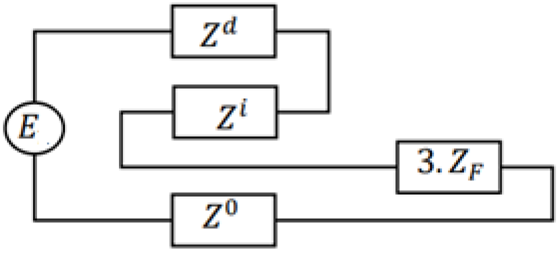


Fig. 13: Connection of the networks for a zero-sequence fault

5.2 Line-to-line fault

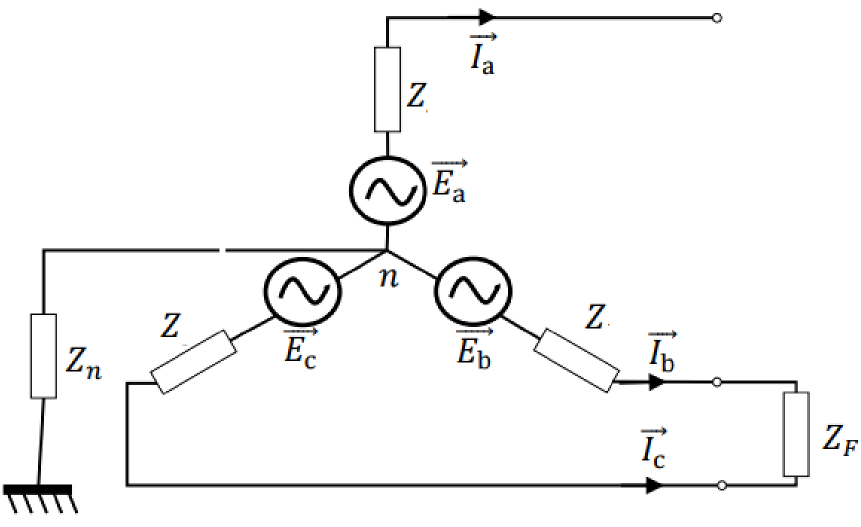


Fig. 14: Line-to-line fault on phases b and c of an unloaded generator

The short-circuit conditions are:

$$I_a = 0\ ;\ I_b = -I_c\ ; \tag{59}$$

$$V_b - V_c = Z_F I_b \tag{60}$$

The symmetrical components of the currents can be calculated by:

$$\begin{bmatrix} I_0 \\ I_d \\ I_i \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \quad (61)$$

That is:

$$I_0 = 0$$
$$I_d = \frac{1}{3} I_b (a - a^2) \quad (62)$$
$$I_i = \frac{1}{3} I_b (a^2 - a) = -I_d$$

Substituting into the voltage equation (60):

$$V_b - V_c = (a^2 V_d + aV_i + V_0) - (aV_d + a^2 V_i + V_0) = (a^2 - a)V_d + (a - a^2)V_i$$
$$= (a^2 - a)(E - I_d Z_d) + (a - a^2)(-I_i Z_i) = (a^2 - a)(E - I_d Z_d) + (a - a^2)(I_d Z_i) \quad (63)$$
$$= (a^2 - a)(E - I_d(Z_d + Z_i))$$

On the other hand :

$$V_b - V_c = Z_F I_b = Z_F (a^2 - a) I_d \quad (64)$$

By comparison:

$$(a^2 - a)(E - I_d(Z_d + Z_i)) = Z_F (a^2 - a) I_d$$
$$a^2 I_d Z_d - a I_i Z_i = aE - a I_d Z_d - a^2 I_i Z_i$$
$$E(\, a^2 - a) = I_d [Z_d (a^2 - a) + Z_i (a^2 - a) + Z_F (a^2 - a)]$$

After simplification :

$$I_d = \frac{E}{Z_d + Z_i + Z_F} \quad (65)$$

The fault current is therefore:

$$I_f = I_b = (a^2 - a) I_d = \frac{(a^2 - a)E}{Z_d + Z_i + Z_F} \quad (66)$$

Whose magnitude is:

$$|I_f| = \frac{\sqrt{3} \cdot E}{|Z_d + Z_i + Z_F|} \quad (67)$$

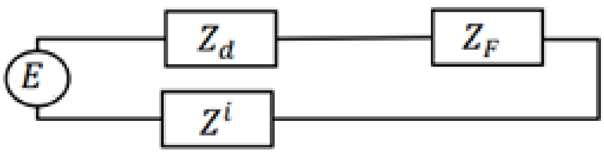


Fig. 15: Connection of the networks for a line-to-line fault

5.3 Double line-to-ground fault

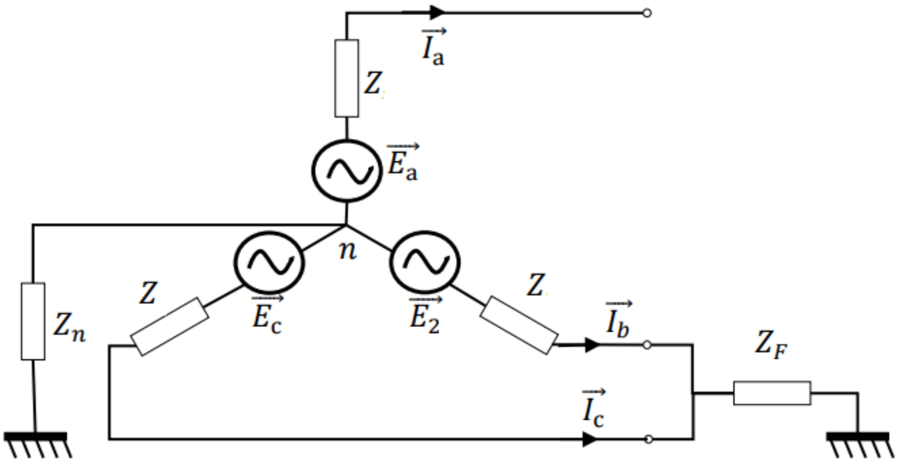


Fig. 16: Double line-to-ground fault on phases b and c of an unloaded generator

The short-circuit conditions are:

$$I_a = 0 \tag{68}$$

$$V_b = V_c = Z_F(I_b + I_c) \tag{69}$$

The symmetrical components of these currents can be calculated as:

$$\begin{bmatrix} I_0 \\ I_d \\ I_i \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ I_c \end{bmatrix} \tag{70}$$

That is :

$$I_0 = \frac{1}{3}(I_b + I_c) \tag{71}$$

$$I_d + I_i = -I_0 \tag{72}$$

Therefore :

$$V_b = Z_F(3I_0) \tag{73}$$

$$V_d = V_i \tag{74}$$

The voltage equation:

$$\begin{gathered} V_b = a^2V_d + aV_i + V_0 = -V_d + V_0 = -(E - I_dZ_d) - I_0Z_0 = 3Z_FI_0 \\ -E + I_dZ_d - I_0(Z_0 + 3Z_F) = 0 \end{gathered} \tag{75}$$

Substituting (72) into (75) :

$$-E + I_dZ_d + (I_d + I_i)(Z_0 + 3Z_F) = 0 \tag{76}$$

Therefore :

$$E = I_d(Z_d + Z_0 + 3Z_F) + I_i(Z_0 + 3Z_F) \tag{77}$$

Now

$$V_d = V_i \;\; \rightarrow \;\; E - I_d Z_d = -I_i Z_i \tag{78}$$

Giving:

$$I_i = \frac{E - I_d Z_d}{-Z_i} \tag{79}$$

Which is substituted into (77):

$$E = I_d(Z_d + Z_0 + 3Z_F) - \frac{E - I_d Z_d}{Z_i}(Z_0 + 3Z_F) \tag{80}$$

Which gives:

$$I_d = \frac{E}{Z_d + \frac{Z_i(Z_0 + 3Z_F)}{Z_i + Z_0 + 3Z_F}} \tag{81}$$

$$I_i = \frac{-E}{Z_i + Z_d + \frac{Z_d Z_i}{Z_0 + 3Z_F}} \tag{82}$$

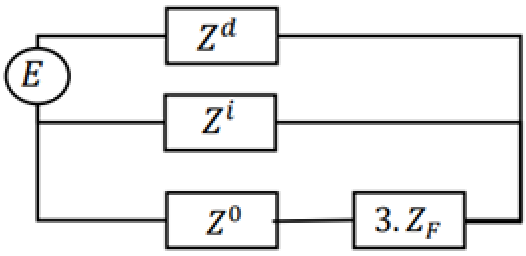
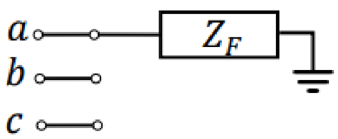
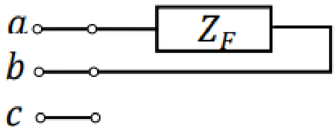
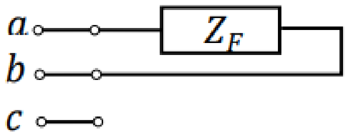


Fig. 17: Connection of the networks for a double line-to-ground fault

5.4 Analysis of asymmetrical faults using the impedance matrix

When the network is balanced, the symmetrical-component impedances are diagonal, so it is possible to calculate Z_{bus} separately for the zero-, positive-, and negative-sequence networks. To obtain a solution for unbalanced faults (at node k), the impedance matrix for each network is obtained separately, and then the sequence impedances Z_{kk0}, Z_{kkd} and Z_{kki} are connected together. The fault formulas for the different unsymmetrical faults are summarized below:

Single-phase fault (phase-to-ground)		$I_{k0} = I_{kd} = I_{ki} = \frac{V_k}{Z_{kkd} + Z_{kki} + Z_{kk0} + 3Z_F}$ $I_f = I_k = 3I_{k0}$ $I_k^0 = 0$

Isolated line-to-line fault		$I_{kd} = -I_{ki} = \dfrac{V_k}{Z_{kkd} + Z_{kki} + Z_F}$ $I_{k0} = 0$ $I_f = I_k = \sqrt{3} I_{kd}$
Double line-to-ground fault		$I_{kd} = \dfrac{V_k}{Z_{kkd} + \dfrac{Z_{kki}(Z_{kk0} + 3Z_F)}{Z_{kki} + Z_{kk0} + 3Z_F}}$ $I_{ki} = -\dfrac{V_k - I_{kd} Z_{kkd}}{Z_{kki}}$ $I_{k0} = -\dfrac{V_k - I_{kd} Z_{kkd}}{Z_{kk0} + 3Z_F}$ $I_f = I_k = 3I_{k0}$

Voltages and currents during the fault:

Using the components of the fault current, we obtain the symmetrical components of the nodal voltages during the fault:

$$\begin{cases} V_{j0} = 0 - Z_{jk0} \cdot I_{k0} \\ V_{jd} = V_{jd} - Z_{jkd} \cdot I_{kd} \\ V_{ji} = 0 - Z_{jki} \cdot I_{ki} \end{cases} \quad (83)$$

V_{j0}, V_{jd} and V_{ji} are the components of the voltage at node j.

The phase voltage at node j is calculated by:

$$V_{jabc} = T.V_{jodi} \quad (84)$$

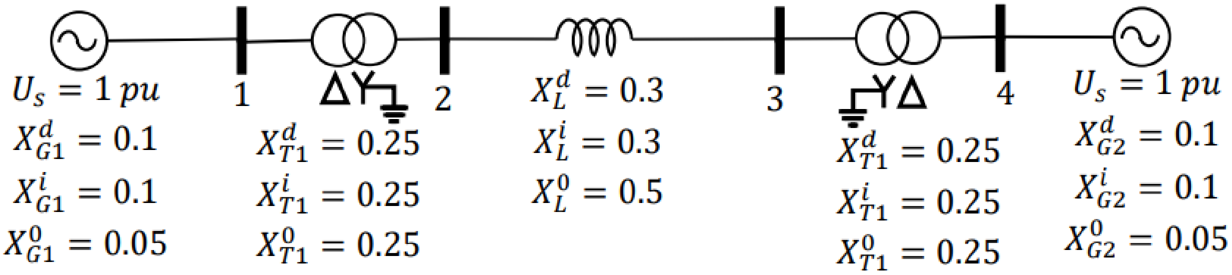
The symmetrical components of the current flowing in line $l - j$ during the fault are given by:

$$\begin{cases} I_{lj0} = \dfrac{V_{l0} - V_{j0}}{z_{lj0}} \\ I_{ljd} = \dfrac{V_{ld} - V_{jd}}{z_{ljd}} \\ I_{lji} = \dfrac{V_{li} - V_{ji}}{z_{lji}} \end{cases} \quad (85)$$

Where z_{lj0}, z_{ljd} and z_{lji} are the zero-, positive-, and negative-sequence components of the impedance of line $l - j$.

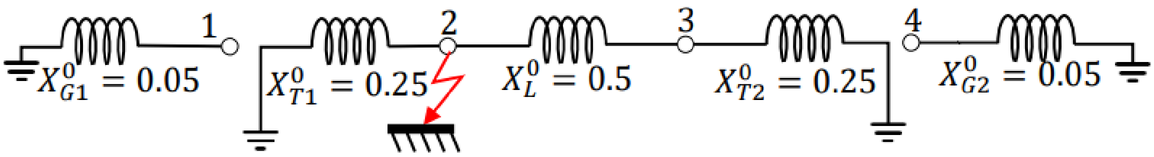
Example 6:

Consider the network in the figure below. Determine the symmetrical and asymmetrical fault currents for a fault at node 2.



Solution :

Positive / negative sequence:

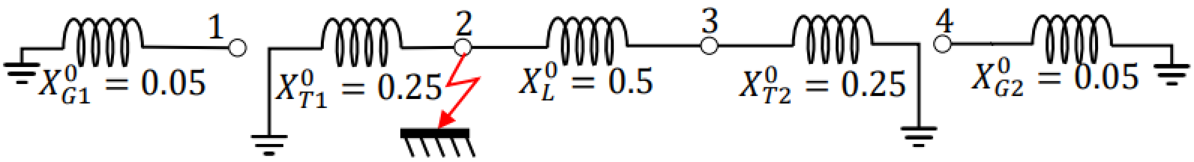


$$Z_{Th}^d = \frac{(Z_{G1}^d + Z_{T1}^d) \cdot (Z_L^d + Z_{T2}^d + Z_{G2}^d)}{Z_{G1}^d + Z_{T1}^d + Z_L^d + Z_{T2}^d + Z_{G2}^d} = \frac{j0{,}35 \times j0{,}65}{j0{,}35 + j0{,}65} = j0{,}2275\,pu$$

The negative-sequence reactances being identical to the positive-sequence reactances:

$$Z_{Th}^i = Z_{Th}^d = j0{,}2275\,pu$$

Zero sequence:



$$Z_{Th}^0 = \frac{Z_{T1}^0 \cdot (Z_L^0 + Z_{T2}^0)}{Z_{T1}^0 + Z_L^0 + Z_{T2}^0} = \frac{j0{,}25 \times j0{,}75}{j0{,}25 + j0{,}75} = j0{,}1875\,pu$$

1) Solid three-phase fault

$$I_{cc2} = \frac{V_{2a}}{Z_{Th}^d} = \frac{1\angle 0°}{j0{,}2275} = -j4{,}3956\,pu$$

2) Single phase-to-ground fault

$$I_{2a}^0 = I_{2a}^d = I_{2a}^i = \frac{V_{2a}}{(Z_{Th}^0 + Z_{Th}^d + Z_{Th}^i)} = \frac{1\angle 0°}{j(0{,}2275 + 0{,}2275 + 0{,}1875)} = -j1{,}5563\,pu$$

$$I_{cc2} = 3I_{2a}^0 = 3 \times (-j1{,}5563) = -j4{,}669\,pu$$

3) Line-to-line fault

$$I_{2a}^{d} = -I_{2a}^{i} = \frac{V_{2a}}{\left(Z_{Th}^{d} + Z_{Th}^{i}\right)} = \frac{1\angle 0°}{j(0,2275 + 0,2275)} = -j2,1978\,pu$$

$$I_{cc2} = \sqrt{3} \cdot I_{2a}^{0} = \sqrt{3} \times (-j2,1978) = -j3,8067\,pu$$

4) Double line-to-ground fault

$$I_{2a}^{d} = \frac{V_{2a}}{Z_{Th}^{d} + \dfrac{Z_{Th}^{i} \cdot Z_{Th}^{0}}{\left(Z_{Th}^{i} + Z_{Th}^{0}\right)}} = \frac{1\angle 0°}{j0,2275 + \dfrac{j0,2275 \times j0,1875}{(j0,2275 + j0,1875)}} = -j3,0267\,pu$$

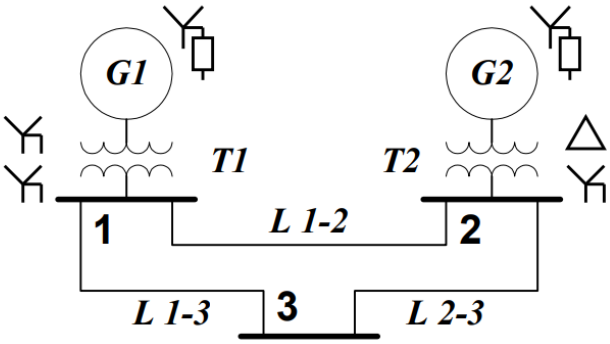
$$I_{2a}^{0} = -\frac{V_{2a} - Z_{Th}^{d} \cdot I_{2a}^{d}}{Z_{Th}^{0}} = -\frac{1\angle 0° - j0,2275 \times (-j3,02767)}{j0,1875} = j1,65975\,pu$$

$$I_{cc2} = 3I_{2a}^{0} = 3 \times (j1,65975) = j4,979\,pu$$

Example 7:

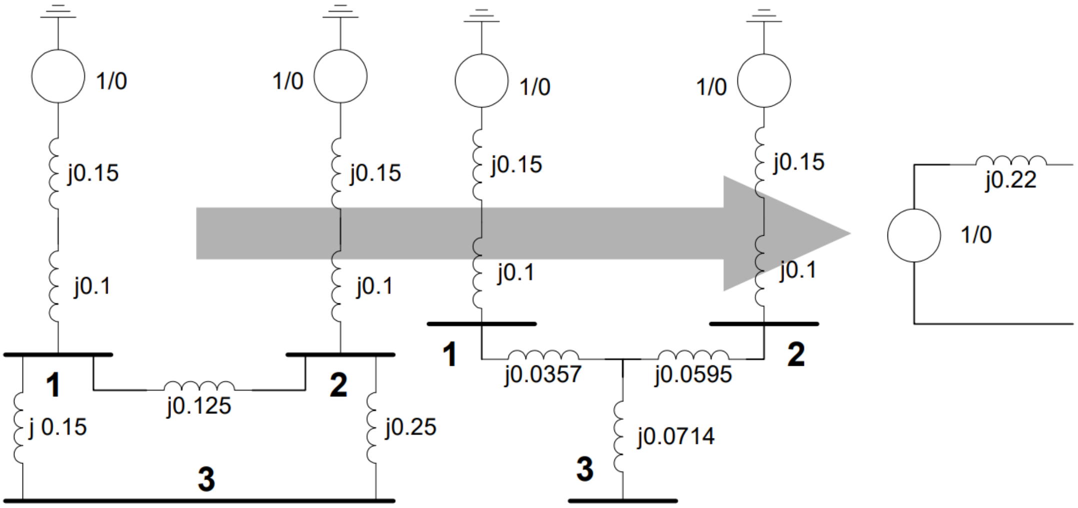
Consider the network in the figure below. The neutral of each generator is grounded through a limiting resistance of 8,333 % on the base 100 MVA. The generators operate at no load at rated voltage and in phase. All data are expressed on the base 100 MVA.

Find the fault currents for the following faults: three-phase, single-phase, line-to-line, and double line-to-ground.

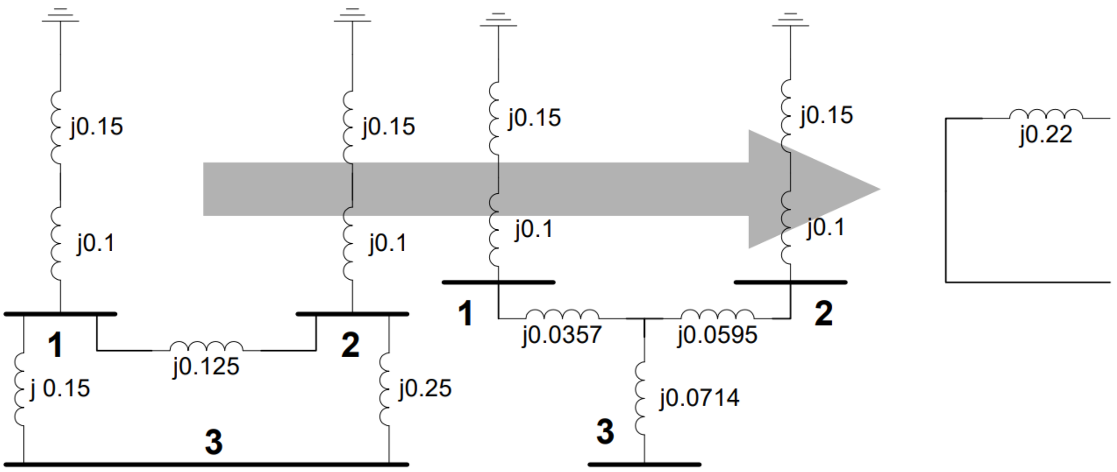


Item	Nominal voltage (kV)	X_d (%)	X_i (%)	X_0 (%)
G1	20	15	15	5
G2	20	15	15	5
T1	20/200	10	10	10
T2	20/200	10	10	10
L12	200	12,5	12,5	30
L13	200	15	15	35
L23	200	25	25	71,25

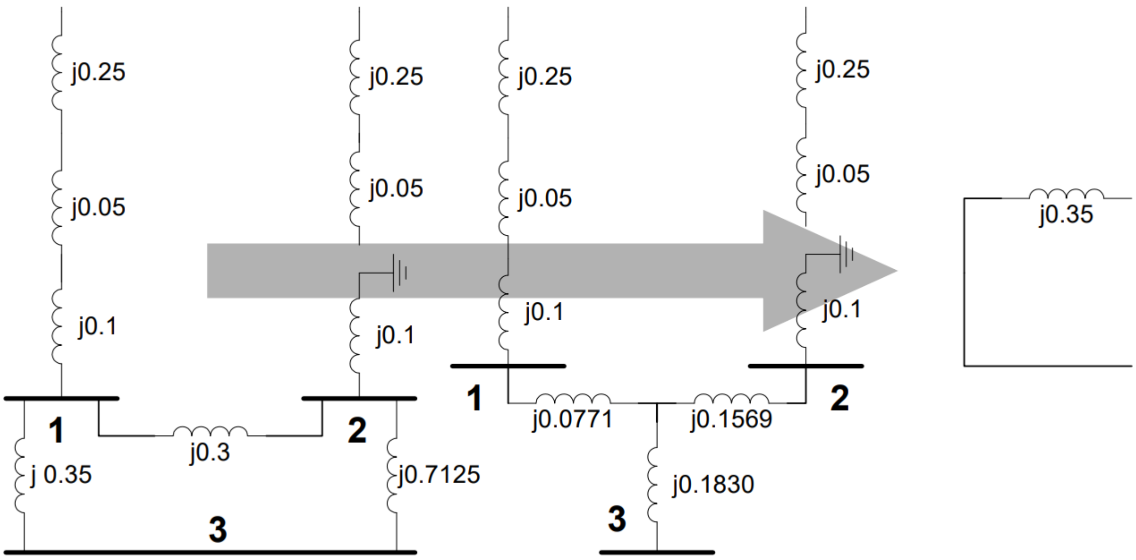
Positive sequence:



Negative sequence:



Zero sequence:



1) Three-phase fault

$$I_{3a}^{d}=\frac{V_{3a}^{d}}{Z_{33}^{d}}=\frac{1\angle 0°}{j0{,}22}=-j4{,}54\,pu$$

$$I_{a}^{f}=I_{3a}^{d}=-j4{,}54\,pu$$

2) Single line-to-ground fault (SLG)

$$I_{3a}^{0}=I_{3a}^{d}=I_{3a}^{i}=\frac{V_{3a}^{d}}{Z_{33}^{0}+Z_{33}^{d}+Z_{33}^{i}}=\frac{1\angle 0°}{j0{,}35+j0{,}22+j0{,}22}=-j1{,}266\,pu$$

$$I_{a}^{f}=3I_{3a}^{d}=-j3{,}80\,pu$$

3) Line-to-line fault (LL)

$$I_{3a}^{0}=0$$

$$I_{3a}^{d}=-I_{3a}^{i}=\frac{V_{3a}^{d}}{Z_{33}^{d}+Z_{33}^{i}}=\frac{1\angle 0°}{j0{,}22+j0{,}22}=-j2{,}27\,pu$$

$$I_{b}^{f}=-I_{c}^{f}=-j\sqrt{3}(-j2{,}27)=-3{,}936\,pu$$

4) Double line-to-ground fault (LLG)

$$I_{3a}^{d}=\frac{V_{3a}^{d}}{Z_{33}^{d}+\frac{Z_{33}^{i}\cdot Z_{33}^{0}}{\left(Z_{33}^{i}+Z_{33}^{0}\right)}}=\frac{1\angle 0°}{j0{,}22+\frac{j0{,}35\times j0{,}22}{(j0{,}35+j0{,}22)}}=-j2{,}816\,pu$$

$$I_{3a}^{i}=-\frac{V_{3a}^{d}-Z_{33}^{d}\cdot I_{3a}^{d}}{Z_{33}^{i}}=-\frac{1\angle 0°-(j0{,}22)\times(-j2{,}816)}{j0{,}22}=j1{,}729\,pu$$

$$I_{3a}^{0}=-\frac{V_{3a}^{d}-Z_{33}^{d}\cdot I_{3a}^{d}}{Z_{33}^{0}}=-\frac{1\angle 0°-(j0{,}22)\times(-j2{,}816)}{j0{,}35}=j1{,}087\,pu$$

$$I_{f}=I_{3b}+I_{3c}=2I_{3a}^{0}+(a+a^{2})\left(I_{3a}^{d}+I_{3a}^{i}\right)=2(j1{,}087)-(-j2{,}816+j1{,}729)=j3{,}261\,pu$$