M1RE//MORE/EXAM24/Corrigé

Exercice 1 (4 pts)

$$S_b = 100\ MVA \qquad U_b = 220\ kV$$

$$Z_b = \frac{U_b^2}{S_b} = \frac{(220)^2}{100} = 484\ \Omega \quad \rightarrow \ Z = \frac{4 + \mathrm{j}60}{484} = 0{,}0083 + \mathrm{j}0{,}124\ \mathrm{pu}$$

$$Y_b = \frac{1}{Z_b} = \frac{1}{484} = 0{,}0021\ S \quad \rightarrow \ Y = \frac{\mathrm{j}2 \cdot 10^{-3}}{0{,}0021} = \mathrm{j}0{,}968\ \ \mathrm{pu}$$

Exercice 3 (6 pts)

$$C_1 = 280 + 5{,}8P_1 + 0{,}003P_1^2 \quad \text{et} \quad C_2 = 150 + 5.4P_2 + 0{,}002P_2^2$$

$$\frac{dC_1}{dP_1} = 5{,}8 + 0{,}006P_1 = \lambda \quad \rightarrow \quad P_1 = \frac{\lambda - 5{,}8}{0{,}006}$$

$$\frac{dC_2}{dP_2} = 5{,}4 + 0{,}004P_1 = \lambda \quad \rightarrow \quad P_2 = \frac{\lambda - 5{,}4}{0{,}004}$$

$$P_L = 0 \ \Rightarrow \ P_1 + P_2 = 420\ MW \ \rightarrow \ \frac{\lambda - 5{,}8}{0{,}006} + \frac{\lambda - 5{,}4}{0{,}004} = 420$$

$$\Rightarrow \quad \lambda = \frac{420 + \frac{5{,}8}{0{,}006} + \frac{5{,}4}{0{,}004}}{\frac{1}{0{,}006} + \frac{1}{0{,}004}} = 6{,}568\ kDA/MWh$$

$$P_1 = \frac{6{,}568 - 5{,}8}{0{,}006} = 128\ MW\ ; \qquad P_2 = \frac{6{,}568 - 5{,}4}{0{,}004} = 420 - 128 = 292\ MW$$

$$P_1 < P_{1min} = 150MW \quad \Rightarrow \quad P_1 = P_{1min} = 150MW$$

$$\rightarrow \ P_2 = 420 - 150 = 270MW$$

$$C_T = C_1 + C_2 = (280 + 5{,}8 \times 150 + 0{,}003 \times 150^2) + (150 + 5{,}4 \times 270 + 0{,}002 \times 270^2)$$
$$= 2971{,}3\ kDA/h$$

Exercice 2 (10 pts)

1) Nœud **1** : **slack** bus ou nœud balancier ; Nœud **2** : nœud **PV** ; Nœud **3** : nœud **PQ**

2)

$$\begin{cases} Y_{ii} = \sum_{j=1}^{3} y_{ij} \\ Y_{ik} = -y_{ik} \quad i \neq k \end{cases} \qquad y_{12} = \frac{1}{j0{,}2} = -j5 \qquad y_{13} = \frac{1}{j0{,}25} = -j4 \quad y_{23} = \frac{1}{j0{,}4} = -j2{,}5$$

$$Y = \begin{bmatrix} -j5 - j4 & j5 & j4 \\ j5 & -j5 - j2{,}5 & j2{,}5 \\ j4 & j2{,}5 & -j4 - j2{,}5 \end{bmatrix} = \begin{bmatrix} -j9 & j5 & j4 \\ j5 & -j7{,}5 & j2{,}5 \\ j4 & j2{,}5 & -j6{,}5 \end{bmatrix}$$

$$Y = G + jB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + j \begin{bmatrix} -9 & 5 & 4 \\ 5 & -7{,}5 & 2{,}5 \\ 4 & 2{,}5 & -6{,}5 \end{bmatrix}$$

3)

$$P_3 = \sum_{k=1}^{3} V_3 V_k [G_{3k} cos(\delta_3 - \delta_k) + B_{3k} sin(\delta_3 - \delta_k)]$$

$$= V_3[B_{31}V_1 sin(\delta_3 - \delta_1) + B_{32}V_2 sin(\delta_3 - \delta_2) + B_{33}V_3 sin(\delta_3 - \delta_3)]$$

$$= V_3[4{,}1 sin(\delta_3) + 2{,}6 sin(\delta_3 - \delta_2)] = -6$$

$$Q_3 = \sum_{k=1}^{3} V_3 V_k [G_{3k} sin(\delta_3 - \delta_k) - B_{3k} cos(\delta_3 - \delta_k)]$$

$$= V_3[-B_{31}V_1 cos(\delta_3 - \delta_1) - B_{32}V_2 cos(\delta_3 - \delta_2) - B_{33}V_3 cos(\delta_3 - \delta_3)]$$

$$= V_3[-4{,}1 cos(\delta_3) - 2{,}6 cos(\delta_3 - \delta_2) + 6{,}5V_3] = -3$$

4) Algorithme de Gauss-Seidel : $\overline{V_i}^{k+1} = \frac{1}{Y_{ii}}\left(\frac{\overline{S_i}^*}{\overline{V_i}^{*k}} - \sum_{j=1}^{i-1} Y_{ij}\overline{V_j}^{k+1} - \sum_{j=i+1}^{n} Y_{ij}\overline{V_j}^{k}\right)$

$\rightarrow$ $\overline{V_2}^{k+1} = \frac{1}{Y_{22}}\left(\frac{\overline{S_2}^*}{\overline{V_2}^{*k}} - Y_{21}\overline{V_1} - Y_{23}\overline{V_3}^{k}\right)$ et $\overline{V_3}^{k+1} = \frac{1}{Y_{33}}\left(\frac{\overline{S_3}^*}{\overline{V_3}^{*k}} - Y_{31}\overline{V_1} - Y_{32}\overline{V_2}^{k+1}\right)$

$$\overline{V}_1 = 1{,}025 + j0\,; \quad \overline{V_2}^0 = 1{,}03 + j0\,; \quad |\overline{V_2}| = 1{,}04; \quad \overline{V_3}^0 = 1{,}0 + j0 \quad pu$$

$$Q_2^1 = V_2[-B_{21}V_1 cos(\delta_2 - \delta_1) - B_{22}V_2 cos(\delta_2 - \delta_2) - B_{23}V_3 cos(\delta_2 - \delta_3)]$$

$$= 1{,}03[-5 \times 1{,}025 cos(0) + 7{,}5 \times 1{,}03(0) - 2{,}5 \times 1 cos(0)] = 0{,}103\, pu$$

$$\rightarrow \quad \overline{S}_2 = 4 + j0{,}103\,; \qquad \overline{S}_3 = -6 - j3\,;$$

$$\overline{V_2}^1 = \frac{1}{-j7{,}5}\left(\frac{4 - j0{,}103}{1{,}03 - j0} - j5 \times 1{,}025 - j2{,}5 \times 1{,}0\right) = 1{,}03 + j0{,}518 = 1{,}153\angle 0{,}466°\, pu$$

$|\overline{V_2}| = 1{,}04$ constante donc $\overline{V_2}^1 = \frac{1{,}153\angle 0{,}466°}{1{,}153} \times 1{,}04 = 1{,}04\angle 0{,}466° = 0{,}929 + j0{,}467\, pu$

$$\overline{V_3}^1 = \frac{1}{-j6{,}5}\left(\frac{-6 + j3}{1{,}0 - j0} - j4 \times 1{,}025 - j2{,}5 \times (0{,}929 + j0{,}467)\right) = 0{,}526 - j0{,}743 = 0{,}910\angle - 0{,}95°\, pu$$

5)

$$\overline{S_1} = P_1 + jQ_1 = \overline{V}_1\left(\sum_{j=1}^{3} Y_{ij}\overline{V_j}\right)^*$$

$$\overline{S_1}^* = P_1 - jQ_1 = \overline{V_1}^* \sum_{j=1}^{3} Y_{ij}\overline{V_j} = \overline{V_1}^*(Y_{11}\overline{V_1} + Y_{12}\overline{V_2} + Y_{13}\overline{V_3})$$

$$= 1{,}025\big(-j9 \times 1{,}025 + j5 \times (1.0399 + j0.0070) + j4 \times (0.9490 - j0.0591)\big)$$

$$= 0{,}2064 - j0{,}2352$$

$P_1 = 0{,}2064\, pu = 206{,}4\, MW$ et $Q_1 = 0{,}2352\, pu = 235{,}2\, MVAr$