Democratic and Popular Republic of Algeria

Ministry of Higher Education and Scientific Research

University of August 20, 1955 - Skikda

Faculty of Technology

Department of Electrical Engineering

Electrotechnical Branch

Course Material

Field Theory

Level : 3rd Bachelor's Degree in Electrical Engineering

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-Lecturer (Grade A) at the University of August 20, 1955 – Skikda-

Academic year 2023/2024

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Chapter VI : Reflection and transmission of electromagnetic waves

Foreword

The handout you have in your hands is primarily intended for 3rd Bachelor's Degree students in Electrical Engineering, those taking the module titled « Field Theory ».

The primary goal of this handout is to equip students with a study resource that facilitates their initial understanding and application of various mathematical tools, such as $\overrightarrow{rot}$, $\overrightarrow{div}$, $\overrightarrow{grad}$, etc., for the calculation of diverse electromagnetic quantities. The intention is to foster a comprehensive comprehension of fundamental concepts, including electric and magnetic fields, different electromagnetism regimes, and the proficient use of Maxwell's equations in calculations related to electromagnetic wave propagation. Furthermore, the course is enriched with an ample number of solved illustrative examples to enhance clarity and reinforce the grasped concepts.

Lastly, I sincerely pray that Allah assists me in imparting the message of knowledge to our esteemed students. I also wish them success and guidance. Amen.

Dr. SEKHANE Hocine

October 2023

Course Syllabus :

« Field Theory ».

١) Course Informations

Faculty : Technology.

Department : Electrical Engineering.

Target Audience : 3rd Bachelor's Degree in Electrical Engineering.

Course Title : Field Theory.

Credits : 04.

Coefficient : 02.

Class Hours : 22 hours and 30 minutes (over 15 weeks).

Instructor : Dr. SEKHANE Hocine.

Contact : Email : docsekhoc@gmail.com

Availability : In the teachers' room between working hours.

٢) Course Overview

The course is divided into six chapters, with each chapter being covered through pedagogical sequences to facilitate the assimilation of the intended concepts.

The management of all course learning units (chapters) is as follows :

Chapter	Hours per Week
I- Electrostatics	3
II- Magnetostatics	3
III- Time-dependent phenomena (Quasi-Stationary Regime)	3
IV- Time-varying regime - Maxwell's equations	3
V- Propagation of the electromagnetic field	2
VI- Reflection and transmission of electromagnetic waves	1

٣) Prerequisites

To fully benefit from this course, it is recommended to have a solid grasp of the following prerequisite knowledge:

- The basic rules of scalar and vector calculus.
- The necessary mathematical tools such as the $\overrightarrow{curl}$, $\overrightarrow{div}$, $\overrightarrow{grad}$, *Laplacian, etc.*

٤) Learning Objectives

The course « Field Theory » aims to :

1- Understanding the fundamental concepts of electromagnetism.

2- Mastering the various characteristics of an electric field and a magnetic field.

3- Adapting the learner to use mathematical tools such as the $\overrightarrow{curl}, \overrightarrow{div}, \overrightarrow{grad}$, etc.

4- Solving various problems in electric and magnetic fields.

5- Assessing various observed electromagnetic phenomena.

٥) Methods of Assessing Learning

The final assessment is determined through two main components:

1) In-class Exam:

This exam covers the entire content taught in the course and contributes 60% to the overall grade.

2) Continuous Assessment:

Throughout the semester, you have the opportunity to accumulate points, which account for 40% of the total grade. This continuous assessment takes various forms, including:

a) The average score of written quizzes.

b) The grade obtained in the group mini-project.

c) The grade for active participation during sessions.

d) Attendance and adherence to discipline.

٦) Supporting Resources

To enrich the learning experience, I provide students with various supporting resources, accessible upon request via email :

- **Reference Documents :** Students can obtain files containing recommended knowledge to deepen their skills in the field.
- **Explanatory Videos :** I also offer explanatory videos and links to online resources that will help students better understand the concepts covered in the course.
- **Tutoring Sessions :** If students require individual assistance, I offer in-person or online tutoring sessions to address their specific questions and help them overcome challenges encountered in the course.
- **Discussion Forums :** Students are encouraged to participate in our online discussion forums where they can engage with their peers and ask questions for a deeper understanding.

I am committed to providing students with a wide range of resources to help them succeed in their learning in this course. Students should not hesitate to use these resources to strengthen their knowledge and understanding of the course.

General Introduction

At the outset, the classical theory of electromagnetism assumed the validity of certain specific mathematical processes, wherein it was deemed possible to localize charge and current distributions within infinitely small volumes of space. This yielded results entirely consistent with experiments on non-atomic scales but contrasting with electromagnetism at the atomic scale, where charges and currents must be described within a non-local quantum formalism [1].

Thereafter, the eminent scholar James Clerk Maxwell systematically unified the two distinct theories of electricity and magnetism into a single theory known as the 'Electromagnetic Theory' or 'Theory of the Electromagnetic Field.' In his research paper titled 'A Dynamical Theory of the Electromagnetic Field,' published in 1865, he also asserted that optics is a subdomain of this theory.

In the early 20th century, Hendrik Antoon Lorentz extended the theory of the electromagnetic field to the microscopic scale and also laid the groundwork for the theory of special relativity, which was fully formulated by Albert Einstein in 1905 [1].

In electromagnetism, depending on the nature of a signal and its propagation velocity, three (03) distinct regimes are distinguished from each other based on their time-dependent variation:

1\ Stationary Regime (SR):

This regime is a phenomenon independent of time (i.e.,$\frac{\partial}{\partial t} = 0$).

Example:

The electric field $\vec{E}$ produced by a stationary and constant electric charge q.

The magnetic field $\vec{H}$ generated by a constant current I.

2\ Quasi-Stationary Regime (QSR):

This regime is a time-varying phenomenon (i.e., $\frac{\partial}{\partial t} \neq 0$) provided that the frequency f<1 kHz (i.e., this regime pertains to low frequencies).

Example: An electrical circuit exhibits a current $I = I_0 \sin(2\pi f t)$ where $f < 1KHz$.

3\ Variable Regime (VR):

This regime is a time-varying phenomenon (i.e., $\frac{\partial}{\partial t} \neq 0$) provided that the frequency f >1 kHz (i.e., this regime pertains to high frequencies).

Example:

Microwave oven, antennas, WiFi, etc.

This course is divided into six (06) chapters, starting with the study of stationary regime in two parts; electrostatics in the first chapter (concepts of electric field, Coulomb's law, Gauss's theorem, etc.), and magnetostatics in the second (concepts of magnetic field, Ampère's theorem, Lorentz force, Laplace force, etc.).

To address time-dependent phenomena, the quasi-stationary regime is organized in the third chapter, covering topics such as Faraday's law, Lenz's law, etc.

The remaining three chapters are dedicated to the study of the variable regime, with the fourth chapter primarily addressing the various integral and differential forms of Maxwell's equations, as well as localized Ohm's law and the boundary conditions of an electromagnetic field. The fifth chapter is intended for the study of the electromagnetic field propagation phenomenon with mathematical and theoretical details. Finally, the last chapter is dedicated to the reflection and transmission of electromagnetic waves in media.

Chapter I: Electrostatics

I.1. Definition of electrostatics

Electrostatics is the branch of physics that studies phenomena created by static electric charges with constant and stationary values for the observer. In other words, no electric current [2].

A simple experiment that anyone can perform to observe electrostatic force involves rubbing a plastic ruler with a dry cloth and bringing it close to small pieces of paper: this is electrification.

A common experience of electrostatic effects is the sensation of receiving a shock when touching a metal object in very dry weather, getting in or out of a car, or removing a synthetic fabric garment. These are phenomena where an accumulation of charges, static electricity, has occurred.

Fig.I.1. Polystyrene balls adhered to a cat's fur due to static electricity.

I.2. Structure of matter

We all know that the materials on our planet are composed of **chemical elements**, such as hydrogen, oxygen, iron, nickel, etc. There are 106 of them in Mendeleev's periodic table of elements. These elements are formed by **atoms**, the basic unit of matter.

The atom consists of a central nucleus composed of protons (positive charges) and neutrons (no charge), surrounded by electrons (negative charges) orbiting around the nucleus. All the mass is concentrated in the nucleus, with electrons having negligible mass.

The atomic mass of an atom is determined by the mass of the nucleus, which is the sum of the number of protons and the number of neutrons. Each atom has an atomic number determined by the number of protons.

If we move up one level in the organization of matter, we have **molecules** formed by an assembly of atoms that are bonded together by two main types of bonds: ionic bonds and covalent bonds.

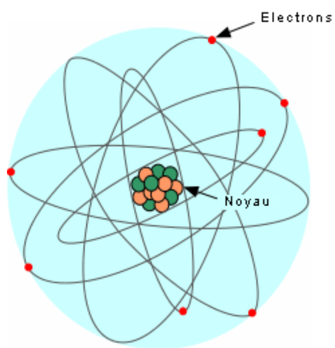
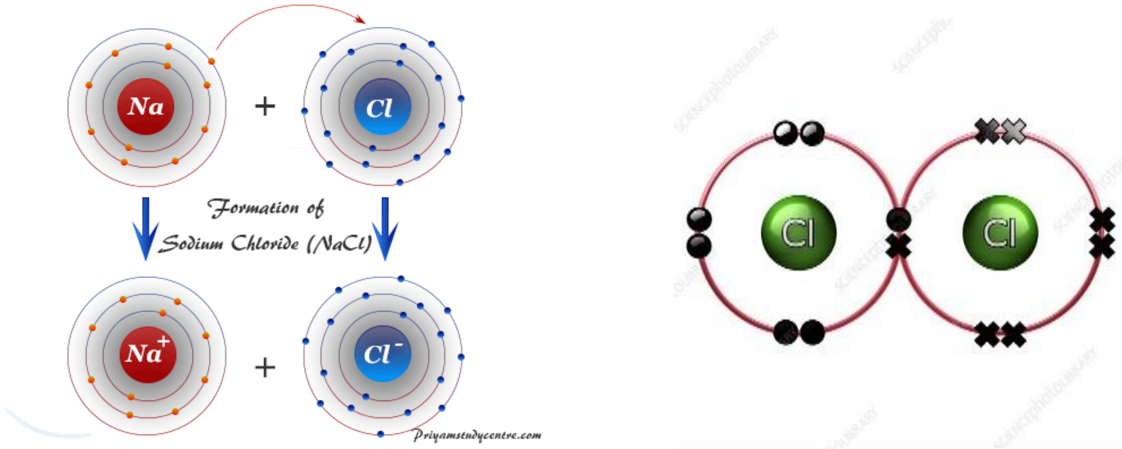


Fig.I.2. Atomic structure

Ionic bonding is facilitated by the transfer of electron(s) from one atom to another. Here, we present the example of NaCl (salt): the transfer of an electron from sodium (Na) to chlorine

(Cl) produces a stable molecule, sodium chloride (NaCl), in which the atoms exist in their ionic form (Na^+ and Cl^- ions).

In covalent bonding, atoms unite through the sharing of electrons. This is the case, for example, with the gases hydrogen (H_2), oxygen (O_2), and chlorine (Cl_2).



(a) **(b)**

Fig.I.3. (a) Ionic bonding ; **(b)** covalent bonding.

I.3. Coulomb's law

I.3.1. Coulomb's law for point charges

Let's consider two point charges, q_1 and q_2, separated by a distance r (see the figure below).

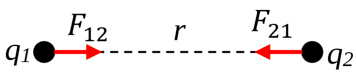


Fig.I.4.

Charge q_1 exerts a force F_{12} on q_2, and similarly, charge q_2 exerts a force F_{21} on q_1.

The Coulomb force is given by:

$$F = F_{12} = F_{21} = \frac{q_1 . q_2}{4\pi\varepsilon_0 r^2} \quad \text{(I.1)}$$

Where:

$F; F_{12}; F_{21}$ in Newtons [N].

$q_1 et\ q_2$ in Coulombs [C].

r in meters [m].

ε_0 is the vacuum (air) permittivity constant, such that $\varepsilon_0 \approx 8.85 * 10^{-12} [\frac{F}{m}]$.

The force is repulsive if the charges have the same sign, and it is attractive if they have opposite signs.

If a charge q is subject to the influence of multiple other charges (n charges), the Coulomb force at this point (charge q) is given by:

$$F(q) = F_1 + F_2 + \cdots + F_n \quad \text{(I.2)}$$

I.3.2. Solved explanatory example (01)

Calculate the attractive force between the electron and proton in the hydrogen atom, given the Bohr radius $r_H = 0.525A^{\circ}$.

Solution :

We have: $1A^{\circ} = 10^{-10}m$. Applying Coulomb's law, we obtain:

$$F_{attr} = F_{ep} = F_{pe} = \frac{é.p}{4\pi\varepsilon_0 r_H^2} = \frac{(-1{,}6.10^{-19}).(1{,}6.10^{-19})}{4\pi(8{,}85.10^{-12})(0{,}525.10^{-10})^2} = |-8{,}35.10^{-8}| = 8{,}35.10^{-8}N$$

I.4. Electric field

I.4.1. Electric field formula

The electric field is a vector quantity created by electrically charged particles and exerts an electric force on other charged particles [3]. Generally, the electric field is given by:

$$E = \frac{F}{q}\ [\text{N/C}]. \quad \text{(I.3)}$$

According to Coulomb's law, we have: $F_{12} = \frac{q_1.q_2}{4\pi\varepsilon_0 r^2}$

The electric field E_1 is created by q_1, but it exerts an electric force F_{12} on the charged particle q_2, thus:

$$E_1 = \frac{F_{12}}{q_2} = \frac{q_1}{4\pi\varepsilon_0 r^2}$$

The electric field E_2 is created by q_2, but it exerts an electric force F_{21} on the charged particle q_1, thus:

$$E_2 = \frac{F_{21}}{q_1} = \frac{q_2}{4\pi\varepsilon_0 r^2}$$

Conclusion : In general, the electric field created by any charge q_n is given by the expression: $E_n = \frac{q_n}{4\pi\varepsilon_0 r^2}$ [N/C], [V/m] (I.4)

When the electric charge q is positive, the direction of the electric field E is outgoing or divergent.

Conversely, when the electric charge q is negative, the direction of the electric field E is incoming or convergent.

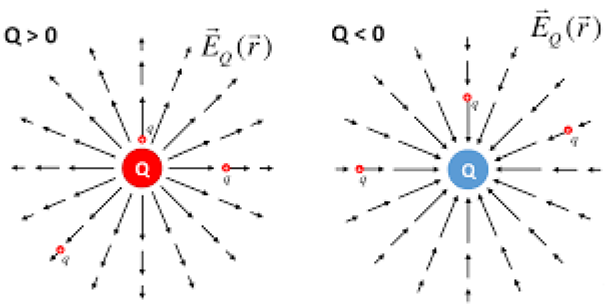


Fig.I.5. Representation of the electric field at various points in space due to a positive elementary charge and another negative charge.

The electric field produced by a set of point charges is equal to the vector sum of the fields produced by all the charges:

$$E = E_1 + E_2 + \cdots + E_n = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \cdots + \frac{q_n}{r_n^2}\right) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2} \quad (I.5)$$

We often adopt an approximate value for $\frac{1}{4\pi\varepsilon_0}$ as approximately 9.10^9

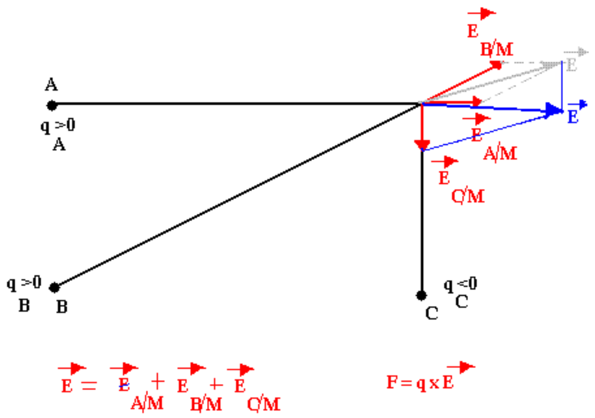
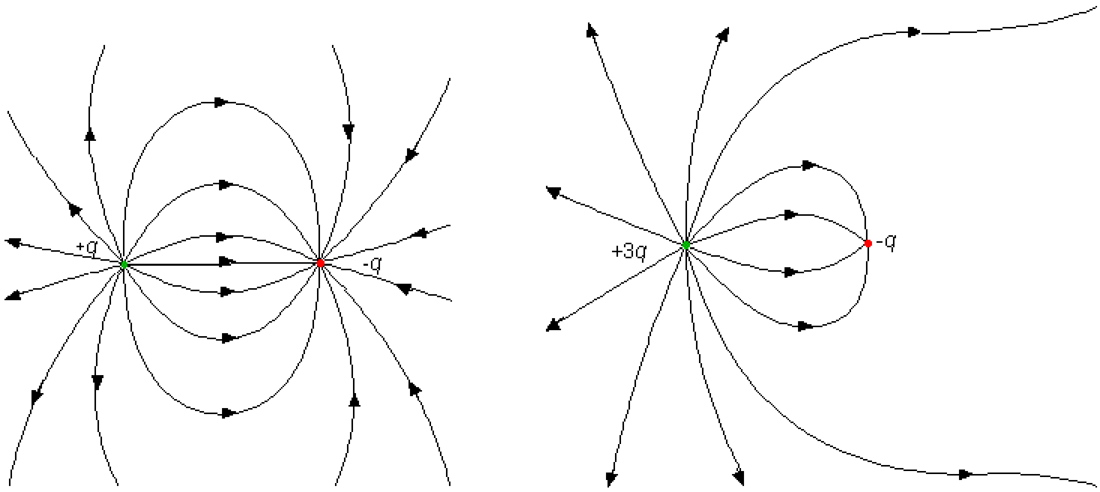


Fig.I.6. Electric field of a set of charges.

I.4.2. Electric field lines.

A static electric field line is defined as an 'oriented curve' in space that is tangent at every point to the associated electric field and oriented in the same direction as the field. In reality, field lines are imaginary constructs.



(a) (b)

Fig.I.7. Representation of electric field lines.

Fig.I.7.(a) : When $|+q| = |-q|$, the number of electric field lines emanating from the positive charge is equal to the number of field lines entering the negative charge.

Fig.I.7.(b) : When $|+q| \neq |-q|$, the number of electric field lines emanating from the positive charge is not equal to the number of field lines entering the negative charge (in our case, the number of outgoing field lines is triple the number of incoming field lines).

(a) **(b)**

Fig.I.8. Representation of the formation of electric field lines.

Fig.I.8.(a) : Two charges of the same sign ⇒ deformation of the electric field lines due to the repulsive force.

Fig.I.8.(b) : Inside a capacitor, the electric field lines are uniform (in reality, there are deformations at the ends of the capacitor, to avoid this, two cores are added).

I.4.3. Solved explanatory example (02)

Consider 4 identical point charges of positive sign placed at the vertices of a square. Determine the value of the electric field at the center of the square.

Solution :

As the 4 charges are of positive sign, the vector of the electric field for each charge should be represented outward from the respective charge and consequently from the center point of the square (see the figure below).

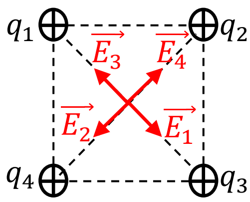


Fig.I.9.

Furthermore, since the 4 charges are identical and at an equal distance from the center of the square, they produce the same magnitude of the electric field, such that:

$\overrightarrow{E_1}$ is opposite to $\overrightarrow{E_3}$, so: $\overrightarrow{E_1} + \overrightarrow{E_3} = 0$; and $\overrightarrow{E_2}$ is opposite to $\overrightarrow{E_4}$, so: $\overrightarrow{E_2} + \overrightarrow{E_4} = 0$

Therefore, the total electric field is: $\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} + \overrightarrow{E_3} + \overrightarrow{E_4} = 0$

I.5. Charge distribution

I.5.1. Charged line

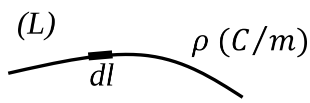


Fig.I.10. Uniformly charged wire

For a uniformly charged wire (see the figure below):

$dq = \rho . dl \Rightarrow q = \int \rho . dl$ where: ρ is the linear charge density (C/m).

I.5.2. Charged surface

For a uniformly charged surface:

$$dq = \rho_s . ds \Rightarrow q = \int_s \rho_s . ds$$

Where : ρ_s is the surface charge density (C/m²).

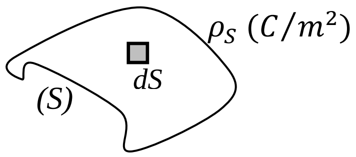


Fig.I.11. Uniformly charged surface.

I.5.3. Charged volume

For a uniformly charged volume:

$$dq = \rho_v . dv \Rightarrow q = \int_v \rho_v . dv$$

Where : ρ_v is the volume charge density (C/m³).

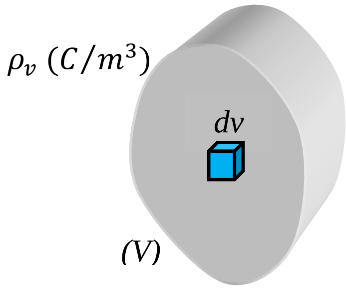
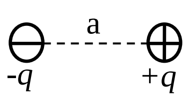


Fig.I.12. Uniformly charged volume

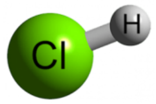
I.6. Electric dipole

The electric dipole (also called an electrostatic dipole) is an arrangement composed of a set of distinct charges arranged in such a way that the center of mass of the positive charges does not coincide with the center of mass of the negatively charged ones, separated by a very small distance, particularly observed at the atomic scale [2, 3].

The simplest dipole consists of two equal and opposite charges separated by a very small distance *a* (see Fig. I.13.(a)). This configuration is observable in certain molecules, such as HCl, for example (see Fig. I.13.(b)).



(a)



(b)

Fig.I.13. (a). Diagram of an electric dipole in its simplest form. **(b)** HCl electric dipole « Hydrogen Chloride ».

The electric dipole moment is given by:

$$\vec{P} = q.\vec{a} \tag{I.6}$$

Where : a is the directed distance from the negative charge to the positive charge.

For a set of *n* charges q_i located at positions $\vec{r_i}$, equation (I.6) becomes:

$$\vec{P} = \sum_{i=1}^{n} q_i \vec{r_i} \tag{I.7}$$

A dipole placed in an electric field is subjected to a torque that tends to align it along the direction of the field.

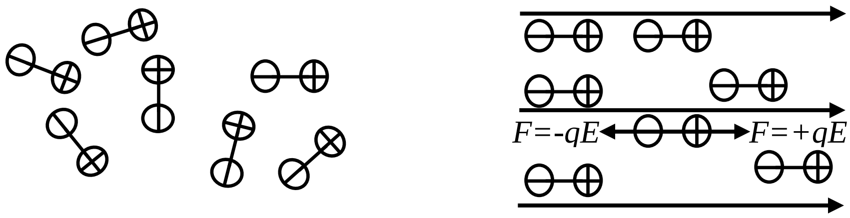


Fig.I.14. Alignment of electric dipoles in the absence and presence of an electric field.

I.7. Electric potential

The expression for the potential created by a charge q is given by:

$$V = E.r = \frac{q}{4\pi\varepsilon_0 r}\ [V], [J/C] \qquad (I.8)$$

The electric potential created by n charges at a point M can be determined from the following expression:

$$V(M) = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \cdots + \frac{q_n}{4\pi\varepsilon_0 r_n} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad (I.9)$$

The potential energy of a charge q subjected to a potential V is given by:

$$E_p = q.V \qquad (I.10)$$

Conclusion : A point charge produces:

- ♣ An electric field (vector quantity): $E = \frac{q}{4\pi\varepsilon_0 r^2}$
- ♣ An electric potential (scalar quantity): $V = \frac{q}{4\pi\varepsilon_0 r}$

I.8. Relationship between the electric field E and the potential V

To place a charge q at a point where a potential V exists, it must supply work $W = -\int F.dr$. This work is stored by the charge q in the form of potential energy $E_p = q.V$. Therefore:

$$W = E_p \Rightarrow dW = dE_p \Rightarrow -F.dr = q.dV \Rightarrow -q.E.dr = q.dV$$

$$\Rightarrow dV = -E.dr \qquad (I.11)$$

On the other hand, we can posit:

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz = \left(\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right)(dx + dy + dz)$$

$$\Leftrightarrow dV = grad\,V.dr \qquad (I.12)$$

From (I.11) and (I.12), we obtain:

$$-E.dr = grad\,V.dr \Rightarrow \boxed{E = -grad\,V} \qquad (I.13)$$

Conclusion : According to this relation (I.13), we conclude that the electric field is always directed from the highest potential to the lowest potential.

According to the useful relationships between operators (see Appendix A), we can write:

$$rot\, E = rot(-\nabla V) = 0 \quad (I.14)$$

Conclusion : According to this relation (I.14), we deduce that the electrostatic field is non-rotational (irrotational), meaning that the electric field lines never form closed loops.

According to equation (I.14): $rot\, E = 0 \Rightarrow \int rot\, E.ds = 0 \Rightarrow \oint E.dl = 0$

Along any closed contour, within which two points A and B are defined:

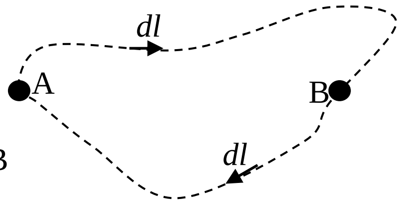


Fig.I.15

$$\oint E.dl = \int_A^B E.dl + \int_B^A E.dl = (V_A - V_B) + (V_B - V_A) = 0 \quad (I.15)$$

I.9. Isopotential Surface

An equipotential surface is a surface where the electric potential is constant and the same everywhere. Examples include a point charge q and a sphere of constant radius. The electric field is always perpendicular to the equipotential surface [3].

I.10. Gauss's Theorem

I.10.1. Electric Flux and Gauss's Theorem

Gauss's theorem allows the calculation of the electric flux through a closed surface, given the electric charges enclosed by it.

An open surface is a surface with a single "opening" defined by a closed contour. One can envision this type of surface as material and rigid; a liquid could not be contained within it as it could escape through the opening. For this surface (example: Fig.I.16.(a) and (b)), the electric flux is:

$$\Phi_e = \int E.dS = \int E.dS.\cos\theta \quad (I.16)$$

A closed surface is a surface without any "opening." One can envision this type of surface as material and rigid; it could contain a liquid without it being able to escape. For this surface (example: Fig.I.16.(c)), the electric flux is:

$$\Phi_e = \int E.dS \quad (I.17)$$

Where :

Total surface area *S*= surface area of the top base S_1 + surface area of the bottom base S_2 + lateral surface area S_3.

So: $$\Phi_e = \int_{S_1} E.dS_1 + \int_{S_2} E.dS_2 + \int_{S_3} E.dS_3 \qquad (I.18)$$

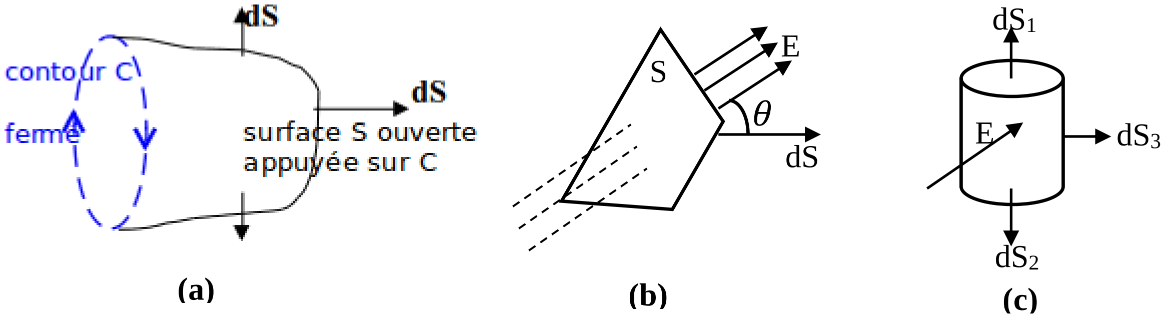


Fig.I.16. (a) (b) Open surfaces ; **(c)** Closed surface.

It is noteworthy that the vectors *dS* associated with the closed surface are perpendicular to the surface under consideration and outward.

On the other hand, when the flux is positive, it is "outward," and when it is negative, the flux is "inward."

To better understand Gauss's theorem, consider multiple charges, some inside a sphere of radius *r* (acting as a closed surface), and others outside (Fig.I.17) [2].

$\Phi_e = \oint E.dS = \oint \frac{q}{4\pi\varepsilon_0 r^2}.dS$; where *r* is constant over the entire surface of the sphere, so we can write: $\Phi_e = \frac{q}{4\pi\varepsilon_0 r^2}\oint dS$; where the surface area of a sphere is $S = 4\pi r^2$, thus:

$\Phi_e = \frac{q}{4\pi\varepsilon_0 r^2}.4\pi r^2$ Therefore: $$\boxed{\Phi_e = \frac{q}{\varepsilon_0}} \qquad (I.19)$$

Charges located outside the closed surface are not considered in Gauss's theorem.

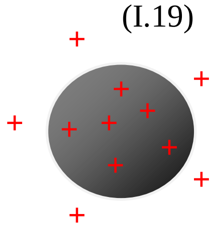


Fig.I.17.

On the other hand, for a differential form, we have $\Phi_e = \int E.dS = \int_v divE dv$, and assuming that the total charge *q* is uniformly distributed within the volume *v*, we can write: $\int_v divE dv = \frac{q}{\varepsilon_0} = \int_v \frac{\rho_v}{\varepsilon_0} dv$, Therefore:

$$\boxed{divE = \frac{\rho_v}{\varepsilon_0}} \qquad (I.20)$$

Conclusion : **(Gauss's Theorem)** The electric flux through any closed surface is equal to the ratio q/ε₀, where *q* represents the sum of charges within that surface.

I.10.2. Solved explanatory example (03)

1) Using Gauss's theorem, determine the expression for the electric field at a point *M* in space produced by an infinite straight charged wire with a linear charge density ρ>0.

2) Deduce the expression for the potential *V* at point *M*.

Solution :

1) We choose the lateral surface of a revolution cylinder around the wire with a chosen height *h* as the Gaussian surface (see the figure opposite).

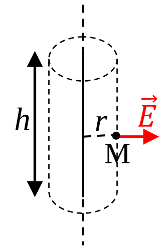


Fig.I.18

Applying Gauss's theorem:

$\Phi_e = \oint E.dS = E.2\pi rh$; Thus: $\Phi_e = \frac{q}{\varepsilon_0} = \frac{\rho h}{\varepsilon_0}$; So, we have: $E.2\pi rh = \frac{\rho h}{\varepsilon_0}$

$$\Longrightarrow E = \frac{\rho}{2\pi\varepsilon_0 r}$$

2) According to the relationship between *E* and *V*:

$$E = -\nabla V \Leftrightarrow E = -\frac{\partial V}{\partial r} \Rightarrow \partial V = -E.\partial r \Rightarrow V = \int -E.\partial r = -\frac{\rho}{2\pi\varepsilon_0 r}\int \frac{1}{r}\partial r$$

$$\Rightarrow V = -\frac{\rho}{2\pi\varepsilon_0}\ln r + cst$$

I.11. Capacitance - Capacitor

In the case of a single conductor subjected to a voltage *V* (see figure opposite):

$$C = \frac{q}{V} \qquad (I.21)$$

Where *C* is the capacitance of the conductor, with its unit [C/V], or Farad [F].

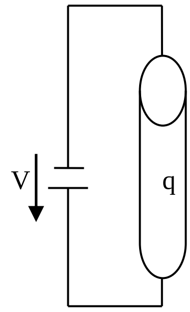


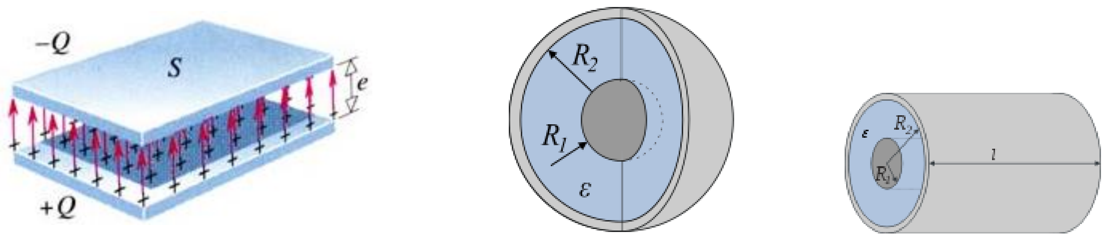
Fig.I.19. Single conductor subjected to a potential difference (voltage).

* **Example:** Charged Sphere (either volumetric or surface charge).

$V = \frac{q}{4\pi\varepsilon_0 r} \Rightarrow C = 4\pi\varepsilon_0 r$

In the case of two conductors forming a capacitor:

$$C = \frac{q}{V} = \frac{q}{|V_1 - V_2|} = \frac{q}{U} \qquad (I.22)$$



(a) **(b)** **(c)**

Fig.I.20. (a) Planar capacitor ; **(b)** Spherical capacitor ; **(c)** Cylindrical capacitor.

I.12. Electrostatic Energy

The electric field stores electric energy with a density:

$$W_e = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{(I.23)}$$

I.13. Interaction between the electric field and matter

I.13.1. For a conductor

As soon as an electric field is applied to a cylindrical conductor placed between two metallic plane plates subjected to a voltage *U*, electrons move under the influence of this field, resulting in a new charge distribution that gives rise to an internal field that cancels the applied field [3].

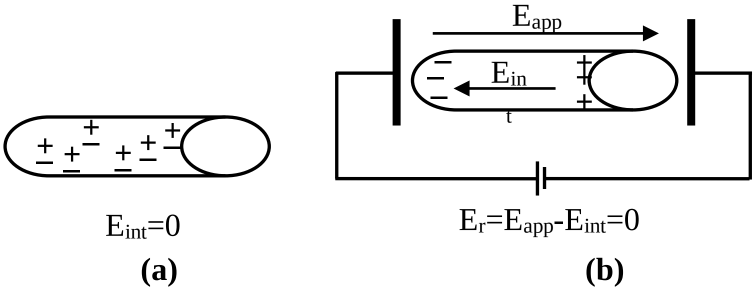


Fig.I.21. (a) In the absence of an electric field ; **(b)** In the presence of an applied electric field.

> ***Conclusion :*** The electric field inside a conductor in electrostatic equilibrium is zero.

I.13.2. For an insulator (dielectric)

The electrons in the insulator are bound to the atoms. When an external electric field is applied, the electrons do not become free but are slightly displaced with respect to the center of gravity of the atom; this is known as polarization [3].

> ***Conclusion :*** The electric field passes through an insulator and becomes zero inside the conductor.

Chapter II : Magnetostatics

II.1. Definition of magnetostatics

Magnetostatics is the study of static magnetic phenomena, i.e., those not dependent on time, generated solely by constant currents (direct current) [3].

II.2. Ampère's law

The Danish physicist Oersted was the first to observe magnetism created by an electric current; he noticed the deviation of a compass placed near a conductor carrying a current (direct current).

Conclusion :

* A stationary electric charge creates only an electric field « **E** ».

* A moving electric charge (current) creates both an electric field « **E** » and a magnetic field « **H** ».

For a straight (infinitely long) conductor, Ampère's Law is valid [4]:

$$H = \int_{-\infty}^{\infty} \frac{Idl \wedge u_r}{4\pi r^2} ; [\frac{A}{m}] \quad \text{(II.1)}$$

*** Magnetic induction « B » (magnetic flux density):** The concentration of magnetic field lines is a measure of density; The greater the field density, the greater the number of lines in the space. The unit of B is Tesla « T » [5].

$$B = \int_{-\infty}^{\infty} \frac{\mu_0 Idl \wedge u_r}{4\pi r^2} ; [T] \quad \text{(II.2)}$$

Where : μ_0 represents the magnetic permeability (of vacuum, air) : $\mu_0 = 4\pi 10^{-7}\ H/m$

So we can write:

$$B = \mu_0 H \quad \text{(II.3)}$$

For a closed conductor: $$B = \oint \frac{\mu_0 Idl \wedge u_r}{4\pi r^2} \quad \text{(II.4)}$$

For a volumetric current (cylindrical) I in a volumetric conductor (cylindrical), let J be the current density [A/m²]:

$$J = \frac{I}{S} \Rightarrow I = JS = \int JdS \Rightarrow Idl = \int JdS\, dl = JSdl = JdV$$

The magnetic field of a volumetric (cylindrical) current is given by:

$$H = \int \frac{J \wedge u_r}{4\pi r^2} dv \quad \text{(II.5)}$$

II.3. Direction of the magnetic field

II.3.1. Right-hand rule

If you hold the conductor in your right hand, with the thumb pointing in the direction of the current, the fingers will point in the direction of the flux. Conversely, if you know the direction of the flux, you can deduce the direction of the current producing it [5].

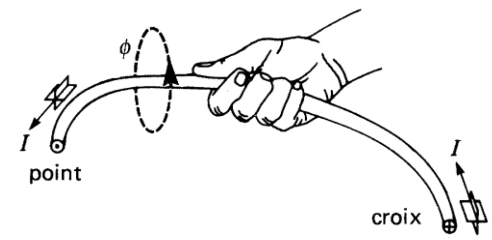


Fig.II.1. Right-hand rule [5].

II.3.2. Resolved explanatory example (04).

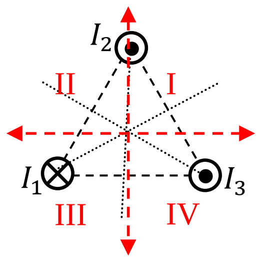


Fig.II.2.

Let's consider three conducting wires arranged in a way that they form an equilateral triangle among them. These three wires carry the same current I (see the figure below).

Determine the trigonometric quadrant in which the vector of the resulting magnetic field is located at the center of the triangle.

<u>Solution :</u>

The application of the right-hand rule for each current *I* involves drawing three circles with their centers at I_1, I_2 *and* I_3.

Thus, we obtain the direction of each of the magnetic fields $\vec{H_1}, \vec{H_2}$ *and* $\vec{H_3}$ as illustrated in the figure below.

By applying the geometric sum method to the vectors $\vec{H_1}, \vec{H_2}$ *and* $\vec{H_3}$ we finally obtain the vector of the total magnetic field $\vec{H}$.

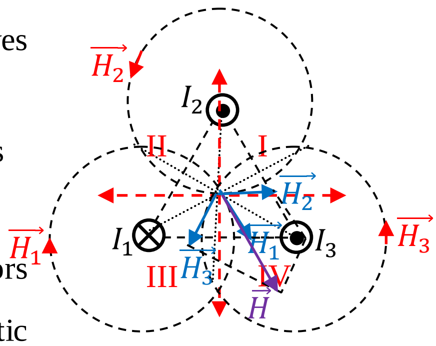


Fig.II.3.

According to the drawing in the figure, it appears that the vector of the resulting magnetic field $\vec{H}$ at the center of the triangle is directed in the trigonometric quadrant No: IV.

II.4. Magnetic potential

The vector magnetic potential ***A*** is given by:

$$A = \frac{\mu_0}{4\pi} \int \frac{J}{r} dv \tag{II.6}$$

Such that the relationship between magnetic induction and magnetic potential is:

$$B = rot\ A \tag{II.7}$$

II.5. Ampère's theorem

We have:

$$H = \frac{I}{2\pi r} u_\theta$$

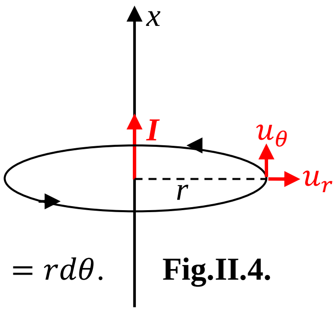


Fig.II.4.

As shown in the figure below, the infinitesimal length *dl* follows u_θ. Since $u_\theta \perp u_r$, that is, $u_\theta \perp r$, it implies that $dl \perp r$.

Therefore, *dl* represents an arc of a circle with a radius of r$\Longrightarrow dl = rd\theta$.

Consequently:

$$\oint H.dl = \frac{I}{2\pi}\oint \frac{rd\theta}{r} = \frac{I}{2\pi}\oint d\theta = \frac{I}{2\pi}[\theta]_0^{2\pi} = I$$

Therefore: $\boxed{\oint H.dl = I}$ (II.8)

Equation (II.8) is the **integral form** of Ampère's theorem.

Note: *I* is a current flowing within the closed contour.

As: $\oint H.dl = \int_s rot\, H.dS$ & $I = \int_s J.dS$ we can write: $\int_s rot\, H.dS = \int_s J.dS$

Hence, the **differential form** of Ampère's theorem: $\boxed{rot\, H = J}$ (II.9)

Conclusion : ***∇×H=J*** implies that the magnetic field is rotational, i.e., the field lines are closed, unlike the electric field lines ($\nabla \times E = 0$).

II.6. Magnetic flux

The magnetic flux is given by:

$$\Phi_m = \int B.dS\, ; [Wb] \text{ (Wiber)} \quad \text{(II.10)}$$

With : $1Wb = 10^8 lines.$

For a non-closed surface, the magnetic flux represents the entirety of magnetic field lines that pass through that surface.

For a closed surface, we have:

$$\oint B.dS = \int divB\, dv = \int div(rot\, A)\, dv = 0$$

Then, the **integral form** of the magnetic flux is: $\boxed{\oint B.dS = 0}$ (II.11)

On the other hand, we have:

$$\oint B.dS = \int divB\, dv = 0 \Longrightarrow \boxed{divB = 0} \quad \text{(II.12)}$$

Equation (II.12) is the **differential form** of the magnetic flux.

II.7. Magnetic force

Two types of forces are distinguished:

II.7.1. Lorentz force

II.7.1.1. Expression of the Lorentz force

An electric charge q moving with a velocity $\vec{v}$ in electric and magnetic fields characterized by vectors $\vec{E}$ & $\vec{B}$ respectively, experiences the following force [4]:

$\vec{F} = q(\vec{E} + \vec{v}\wedge\vec{B}) = q\vec{E} + q\vec{v}\wedge\vec{B} = F_e + F_{mL}$; With :

$$\vec{F_e} = q\vec{E} \quad \text{(II.13)}$$

$$\boxed{\overrightarrow{F_{mL}} = q(\vec{v}\wedge\vec{B})} \quad \text{(II.14)}$$

$\vec{F_e}$: Electric force.

$\overrightarrow{F_{mL}}$: Lorentz magnetic force.

Conclusion : The electric force is null if the charge is zero ($q=0$), and the magnetic force is null if the charge is zero ($q=0$), or if it is stationary ($\vec{v}=0$).

II.7.1.2. Characteristics of the Lorentz force

a) Direction : Perpendicular to $q\vec{v}$ & $\vec{B}$, thus lying in the plane formed by $q\vec{v}$ & $\vec{B}$.

b) Sense : determined either by:

b1) the rule of the three fingers of the right hand: (see Fig.II.5.(a))

The thumb: sense of $q\vec{v}$ (=sense of $\vec{v}$ if q>0 ; = opposite sense to $\vec{v}$ if q<0).

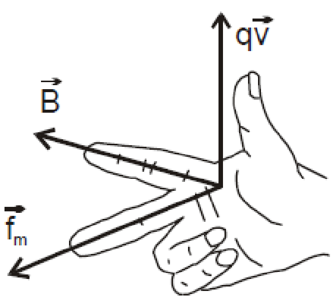
The index finger: sense of $\vec{B}$.

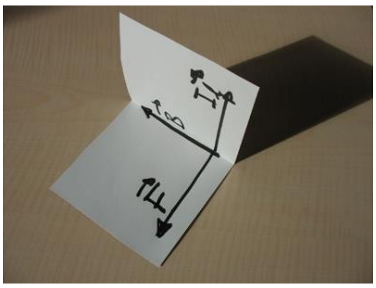
The middle finger: sense of $\overrightarrow{F_{mL}}$.

b2) the folded sheet method: (see Fig.II.5.(b))

c) Magnitude (Intensity): given by:

$$\boxed{F_{mL} = \left| qvB\sin(q\vec{v}\ \hat{;}\ \vec{B}) \right|} \quad \text{(II.15)}$$





(a) **(b)**

Fig.II.5. (a) Right-hand rule with three fingers; **(b)** Folded sheet method.

II.7.1.3. Resolved explanatory example (05)

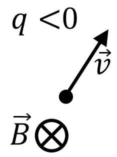


Fig.II.6.

Determine in the illustrated case shown in the figure, the direction, sense, and magnitude of the Lorentz force, if: $v = 2.10^4 m/s$; $B = 0.1T$; $|q| = 1.6\,.\,10^{-19}C$.

Solution :

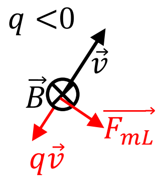


Fig.II.7.

Applying either the right-hand rule or the folded sheet method to Figure (II.6), we obtain the direction and sense of the vectors: $q\vec{v}$ & $\overrightarrow{F_{mL}}$ (Lorentz magnetic force) (see the figure below).

The intensity of the Lorentz force is:

$$F_{mL} = \left|qvB\sin(q\vec{v}\,\hat{;}\,\vec{B})\right| = |-1{,}6\,.\,10^{-19}.\,2.\,10^4.\,0{,}1.\sin(90°)| = 3{,}2.\,10^{-16}N$$

II.7.2. Laplace force

II.7.2.1. Expression of the Laplace force

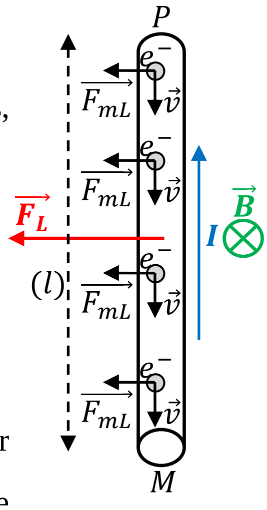


Fig.II.8

We consider a conductor of length l=PM, through which a current I flows, positioned in a magnetic field $\vec{B} \perp PM$ (see the figure opposite).

Every electron moving with a velocity $\vec{v}$ in this conductor experiences the Lorentz force with magnitude:

$$F_{mL} = \left|-evB\sin(q\vec{v}\,\hat{;}\,\vec{B})\right| = evB\sin\alpha$$

For n electrons in the conductor, there will be n Lorentz forces where their resultant $\overrightarrow{F_L}$ represents the electromagnetic force of Laplace acting on the entire conductor, thus:

$$F_L = nF_{mL} = nevB\sin\alpha \qquad \text{(II.16)}$$

Using the right-hand rule or the folded-sheet method, we determine the direction of the Lorentz force (see Fig.II.8).

On the other hand, we have: $I = \frac{q}{t}$; where:

q is the total charge passing through the conductor's cross-sectional area during a duration t.

t is the time it takes for the n electrons present in the conductor to flow through the cross-section at point M (see the figure).

Therefore: $q = ne$ & $t = \frac{l}{v}$; Thus: $I = \frac{nev}{l} \Leftrightarrow Il = nev$

From equation (II.16), we obtain: $F_L = IlB\sin\alpha$ (II.17)

Here, $\alpha = 90° \Longrightarrow \sin\alpha = 1 \Longrightarrow F_L = IlB$

In a general manner, we can express it in integral form: $$F_L = \int Idl \wedge B \qquad \text{(II.18)}$$

II.7.2.2. Characteristics of the Lorentz Force

For a conductor of length *l* placed in a magnetic field and carrying a current *I*, it is subjected to a Lorentz force $\overrightarrow{F_L}$ that has the following characteristics:

a) Direction : Perpendicular to the plane formed by the conductor and $\vec{B}$.

b) Sense : Determined by the right-hand rule or by the folded-sheet method.

c) Norme (Intensity): Given by [4]:

$$F_L = \int Idl \wedge B = IlB \sin\alpha$$

II.8. Magnetic Energy

In air or in any other non-magnetic material, the stored magnetic energy is given by [5]:

$$W_m = \frac{B^2 V}{2\mu_0} \qquad \text{(II.19)}$$

Where: $B = \mu_0 H$ and thus: $$W_m = \frac{{\mu_0}^2 H^2 V}{2\mu_0} = \frac{1}{2}\mu_0 H^2 V \qquad \text{(II.20)}$$

The equation (II.20) represents the total energy stored in the magnetic field *H*. Therefore, the magnetic energy density is:

$$W_m = \frac{1}{2}\mu_0 H^2 \ [\text{J}/m^3] \qquad \text{(II.21)}$$

Chapter III : Time-dependent phenomena (Quasi-Stationary Regime)

III.1. Quasi-Stationary Regime (QSR)

III.1.1. Definition

As previously explained in the general introduction, it is reiterated that the quasi-steady-state regime (QSR) is a time-varying phenomenon (c.à.d. $\frac{\partial}{\partial t} \neq 0$) under the condition that the frequency $f < 1\ KHz$ (i.e., this regime pertains to low frequencies) or the wavelength λ of the signal is much larger than the length of the circuit.

Example: An electrical circuit with a current $I = I_0 \sin(2\pi f t)$ that varies slowly ($f < 1\ KHz$). In this case, the signal propagation phenomenon in the circuit is neglected, and one can apply Ampère's law, Ohm's law, Kirchhoff's laws, Gauss's laws, etc.

III.1.2. Solved Explanatory Example (06)

Check if we can use the quasi-steady-state approximation for the following two cases:

Case 01 : A power transmission line with a frequency *f*=60 Hz, a length *l*= 80Km, and a sinusoidal voltage.

Case 02 : An electrical circuit with a frequency of 2 GHz and a length *l*=15 Cm.

<u>Solution :</u>

Case 01 : First, we calculate the wavelength of the voltage using the formula $\lambda = \frac{v}{f}$, where v is the propagation speed (considered as the speed of light). The wavelength $\lambda = \frac{3*10^8}{60} = 5000\ Km \gg l \Rightarrow$ we can use the quasi-steady-state approximation.

Case 02 : $\lambda = \frac{3*10^8}{2*10^9} = 15\ cm = l \Rightarrow$ we cannot use the quasi-steady-state approximation, and we must consider the propagation phenomena of signals within this circuit.

III.2. Faraday's Law

Consider a closed surface (S) bounded by a closed contour (C) (loop). If the magnetic flux Φ through (S) varies with time, then there exists an induced voltage (or electromotive force, EMF) (e) around (C) given by [6, 7]:

$$\boxed{e = -\frac{d\Phi}{dt};\ [V]} \quad \text{(III.1)}$$

Thus, an induced current (*i*) flows in the closed circuit:

$$\boxed{i = \frac{e}{r};\ [A]} \quad \text{(III.2)}$$

Where: r is the resistance of the coil (S) in ohms [Ω].

Consider the coil located at a distance of *x* from a straight conductor carrying a current *I*:

a) Variable Current *I* : Assuming a variable current (for example, $I = I_0 sin\omega t$), the magnetic induction B at any point M (see Fig.III.1. below) is given by: $B = \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 I_0 sin\omega t}{2\pi x}$

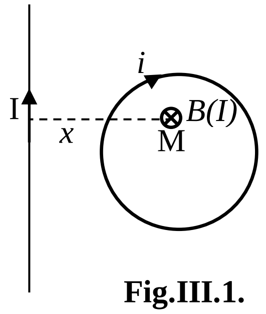


Fig.III.1.

As B is variable, the flux $\Phi = \int B.dS$ is also variable, and thus $\frac{d\Phi}{dt} \neq 0$. This variation in flux generates an electromotive force (EMF) *e* and an induced current *i* in the coil.

b) Constant Current *I* : If the current *I* is constant, then the induction *B* is constant as well: $B = \frac{\mu_0 I}{2\pi x}$. In this case, the flux is constant, so $\frac{d\Phi}{dt} = 0 \Longrightarrow e = 0 \Longrightarrow i = 0$.

c) Constant Current *I* and Coil Movement: If the current *I* is constant and the coil is moving (away or towards) from the constant current *I* at a constant velocity *v* (see Fig.III.2. below), the induction *B* becomes variable (as *x* is variable) ⟹ Φ is variable, inducing an electromotive force (EMF) *e* and a current *i* in the coil.

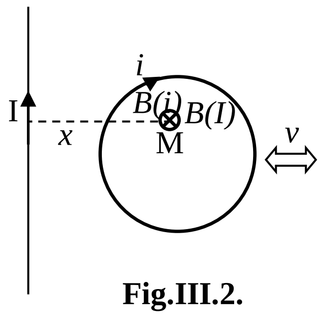


Fig.III.2.

III.3. Lenz's Law

III.3.1. Law concept and "Minus" sign meaning

> **Lenz's Law :** *« The induced voltage (e) by a changing flux ($\frac{d\Phi}{dt} \neq 0$), has a polarity such that the induced current (i) established in a closed loop results in a self-induced magnetic flux (B(i)) that opposes the change in the main flux B(I). »*

Consider the coil from (Fig.III.2). The main induction *B(I)* has an inward sense (as per the right-hand rule).

* If the coil moves away from the current *I* ⟹ *B(I)* decreases ⟹ Φ decreases ⟹ (variation ($\frac{d\Phi}{dt}$) decreases) ⟶ **Applying Lenz's Law:** The self-induced induction *B(i)* of the induced current opposes this change ⟹ $B(i)$ must increase the variation ($\frac{d\Phi}{dt}$) ⟹ $B(i)$ must increase Φ ⟹ *B(i)* must have the same sense (inward) as the main induction *B(I)* so that the resulting induction in the coil increases ($B_r = B(I) + B(i)$) ⟹ Since *B(i)* has an inward sense, the induced current *i* circulates clockwise (Applying the right-hand rule).

* If the coil approaches the current *I* ⟹ *B(I)* increases ⟹ Φ increases ⟹ (variation ($\frac{d\Phi}{dt}$) increases) ⟶ **Applying Lenz's Law:** The self-induced induction B(i) of the induced current

opposes this change $\Longrightarrow B(i)$ must decrease $\Phi \Longrightarrow$ *B(i)* must have the opposite sense to the main induction *B(I)* so that the resulting induction in the coil decreases ($B_r = B(I) - B(i)$) $\Longrightarrow$ Since *B(i)* has an outward sense, the induced current *i* circulates counterclockwise (Applying the right-hand rule).

Note: To fully understand, refer to Appendix B (taken from the renowned book « Electrotechnique » by Théodore Wildi).

We can define the induced electromotive force (e) as the circulation of an electromotive field $\overrightarrow{E_m}$ around the closed circuit (coil) of length *l*. Thus, it can be expressed as follows:

$$e = -\frac{d\Phi}{dt} = \oint \overrightarrow{E_m}\,\overrightarrow{dl} \qquad \text{(III.3)}$$

This force finds its application in induction motors, induction furnaces, vehicle retarders, and automobile clutches [6].

III.3.2. Solved Explanatory Example (07)

Demonstrate that the electromotive field $\overrightarrow{E_m}$ is the derivative of the magnetic vector potential $\vec{A}$.

Solution :

According to (III.3) : $e = -\frac{d\Phi}{dt} = \oint \overrightarrow{E_m}\,\overrightarrow{dl}$

We have: $\Phi = \oiint \vec{B}\,\overrightarrow{dS}$; $\vec{B} = \overrightarrow{curl}\vec{A}$ et $\oint \overrightarrow{E_m}\,\overrightarrow{dl} = \oiint \overrightarrow{curl}\overrightarrow{E_m}\,\overrightarrow{dS}$

Therefore:

$e = -\oiint \frac{d}{dt}\vec{B}\,\overrightarrow{dS} = -\oiint \frac{d}{dt}\overrightarrow{curl}\vec{A}\,\overrightarrow{dS} = \oiint \overrightarrow{curl}\overrightarrow{E_m}\,\overrightarrow{dS} \Longrightarrow \overrightarrow{curl}\left(-\frac{d\vec{A}}{dt}\right) = \overrightarrow{curl}\overrightarrow{E_m} \Longrightarrow \overrightarrow{E_m} = -\frac{d\vec{A}}{dt}$ (Q.E.D.).

III.4. Integral and differential form

III.4.1. Integral form

We studied in Chapter I that the potential difference across:

<u>* An open contour (unclosed loop, open coil)</u> is given by: $U_{AB} = V_A - V_B = \int_A^B E.dl$ or alternatively: $U_{BA} = V_B - V_A = \int_B^A E.dl$

<u>* A closed contour (closed loop, closed coil)</u> is: $U = \int_A^B E.dl + \int_B^A E.dl = (V_A - V_B) + (V_B - V_A) = \oint E.dl = 0 \Longrightarrow$ the voltage in a closed coil is zero.

Consequently, Faraday's law can be expressed in the following integral form:

$e = -\frac{d\Phi}{dt} = \oint E.dl$ And, since $\Phi = \int B.dS \Longrightarrow$ $\boxed{\oint E.dl = -\frac{d}{dt}\int B.dS}$ (III.4)

III.4.2. Differential form

The integral for a closed loop $\oint E.dl$ can be transformed into a surface integral: $\oint E.dl = \int_S curl\, E.dS$. Therefore, from the integral form, we obtain: $\int_S curl\, E.dS = \int -\frac{dB}{dt}.dS$

The differential form of Faraday's law is therefore: $\boxed{curl\, E = -\frac{dB}{dt}}$ (III.5)

Conclusion : According to equation (III.4), we conclude that a variable magnetic field ($\frac{dB}{dt}$) creates an electric field E. This electric field is responsible for the induced current.

* In **RS** : $curl\, E = 0 \Longrightarrow$ E is non-rotational (irrotational) (does not circulate).

* In **RQS** : $curl\, E = -\frac{dB}{dt} \Longrightarrow$ E is rotational (circulates).

III.5. Comparison between Steady State Regime (SSR) and Quasi-Steady State Regime (QSSR)

III.5.1. Comparison Table

The following table provides a summary comparison between the two states SSR and QSSR in terms of the equations used, either in their integral or differential forms:

Table.III.1. Comparison between SSR and QSSR.

SSR	QSSR	SSR & QSSR (common)
$\oint E.dl = 0$	$\oint E.dl = -\frac{d}{dt}\int B.dS$	$\oint E.dS = \frac{q}{\varepsilon}$
$curl\, E = 0$	$curl\, E = -\frac{dB}{dt}$	$\oint H.dl = I$
$E = -grad\, V$	$E = -grad\, V - \frac{dA}{dt}$	$\oint B.dS = 0$

III.5.2. Solved Explanatory Example (08)

In SSR : $E = -grad\, V$; demonstrate that in QSSR : $E = -grad\, V - \frac{\partial A}{\partial t}$, where A is the magnetic vector potential.

Solution :

In QSSR : $\overrightarrow{curl}\, \vec{E} = -\frac{d\vec{B}}{dt}$, where $\vec{B} = \overrightarrow{curl}\, \vec{A}$

Therefore: $\overrightarrow{curl}\,\vec{E} = -\frac{d}{dt}\left(\overrightarrow{curl}\,\vec{A}\right) = -\overrightarrow{curl}\frac{d\vec{A}}{dt} \Longrightarrow \overrightarrow{curl}\,\vec{E} + \overrightarrow{curl}\frac{d\vec{A}}{dt} = 0 \Longrightarrow \overrightarrow{curl}\left(\vec{E} + \frac{d\vec{A}}{dt}\right) = 0$

In SSR : $\overrightarrow{curl}\,\vec{E} = 0 \Longrightarrow \vec{E} = -\overrightarrow{grad}\,\vec{V}$

By analogy with SSR: $\overrightarrow{curl}\left(\vec{E} + \frac{d\vec{A}}{dt}\right) = 0 \Longrightarrow \vec{E} + \frac{d\vec{A}}{dt} = -\overrightarrow{grad}\,\vec{V} \Longrightarrow \vec{E} = -\overrightarrow{grad}\,\vec{V} - \frac{d\vec{A}}{dt}$ (Q.E.D).

Chapter IV : Time-varying regime - Maxwell's equations

IV.1. Principle of charge conservation

IV.1.1. Integral form

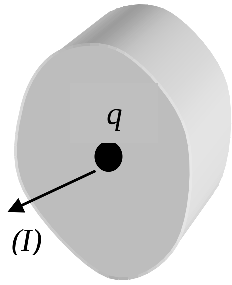


Fig.IV.1

Consider an electric charge q inside any closed surface, and a current I **leaving** (see the figure below).

The principle of charge conservation implies that the current I leaving S $\Leftrightarrow$ decrease in q inside S, thus: $I = -\frac{dq}{dt}$

We have $I = \oint J.dS = -\frac{dq}{dt}$

Furthermore, according to Gauss's theorem: $\oint E.dS = \frac{q}{\varepsilon} \Longrightarrow q = \varepsilon \oint E.dS$

Therefore: $\oint J.dS = -\frac{d}{dt}(\varepsilon \oint E.dS) \Leftrightarrow \oint J.dS + \oint \varepsilon \frac{dE}{dt}.dS = 0$

$$\Leftrightarrow \boxed{\oint \left(J + \varepsilon \frac{dE}{dt}\right).dS = 0} \quad \text{(IV.1)}$$

Equation (IV.1) represents the integral form of the charge conservation equation.

IV.1.2. Differential form

By transposing the previous surface integral into a volume integral, we obtain: $\oint \left(J + \varepsilon \frac{dE}{dt}\right).dS = \oint_V div\left(J + \varepsilon \frac{dE}{dt}\right)dv = 0 \Longrightarrow div\left(J + \varepsilon \frac{dE}{dt}\right)dv = 0$; Knowing that: $div\, E = \frac{\rho}{\varepsilon}$ (see equation (I.20)).

Therefore: $div\left(J + \varepsilon \frac{dE}{dt}\right)dv = div\, J + \varepsilon \frac{d}{dt}(div\, E) = 0 \Longrightarrow$ $\boxed{div\, J + \frac{d\rho}{dt} = 0}$ (IV.2)

Equation (IV.2) represents the differential form of the charge conservation equation [8].

Note: In RS and RQS, $\frac{d\rho}{dt} \approx 0$ & $\frac{dE}{dt} \approx 0$, so they are only considered in the RV. The charge conservation equation in these cases (RS and RQS) becomes: $\oint J.dS = 0$ or $div\, J = 0$

IV.2. Maxwell's Ampere's Law

IV.2.1. Integral form

In RS and RQS, according to Ampère's theorem: $\oint H.dl = I$ where $I = \oint J.dS \Longrightarrow \oint H.dl = \oint J.dS = 0$

But in RV according to equation (IV.1): $\oint \left(J + \varepsilon \frac{dE}{dt}\right).dS = 0$

By analogy, we obtain the integral form of Maxwell's Ampere's Law:

$$\boxed{\oint H.dl = \oint \left(J + \varepsilon \frac{dE}{dt}\right).dS} \quad \text{(IV.3)}$$

IV.2.2. Differential form

In RS and RQS, $div\, J = 0 \Leftrightarrow rot\, H = J$ (equation (II.9) of the differential form of Ampère's theorem).

By analogy in RV: $div\,\left(J + \varepsilon \frac{dE}{dt}\right) = 0 \Leftrightarrow$ $\boxed{rot\, H = J + \varepsilon \frac{dE}{dt}}$ (IV.4)

Equation (IV.4) is the differential form of Maxwell's Ampere's Law.

Conclusion : For the RV, Maxwell added the term $\varepsilon \frac{\partial E}{\partial t}$, and in this case, Ampere's theorem becomes Maxwell's Ampere's Law.

IV.3. Maxwell's equations

IV.3.1. Maxwell's equations and their meanings

In electromagnetism, there are four fundamental equations established by Maxwell, which are:

IV.3.1.1. Maxwell's-Gauss's equation (MG)

a) Integral form : $\oint E.dS = \frac{q}{\varepsilon}$ signifies that the electric flux passing through a closed surface is equal to the ratio $\frac{q}{\varepsilon}$.

b) Differential form : $div\, E = \frac{\rho}{\varepsilon}$ signifies that the electric charge is the source of the electric field [8].

IV.3.1.2. Maxwell's-Magnetic flux equation (M*Φ*)

a) Integral form : $\oint B.dS = 0$ signifies that the magnetic flux passing through a closed surface is zero.

b) Differential form : $div\, B = 0$ by analogy with the MG equation, this equation signifies that there is no "magnetic charge" in nature [8, 9].

IV.3.1.3. Maxwell's-Faraday's equation (MF)

a) Integral form : $\oint E.dl = -\frac{\partial}{\partial t}\int B.dS$ signifies that a conductor traversed by a variable magnetic flux is the seat of an induced electromotive force (emf).

b) Differential form : $rot\, E = -\frac{\partial B}{\partial t}$ signifies that a variable magnetic field ($\frac{dB}{dt}$) creates a variable electric field E [7, 8].

IV.3.1.4. Maxwell's-Ampère's equation (MA)

a) Integral form : $\oint H.dl = \oint \left(J + \varepsilon \frac{dE}{dt}\right).dS$ signifies that the circulation of the magnetic field around a closed loop is equal to the flux of current through a surface (s) bounded by that loop.

b) Differential form : $rot\ H = J + \varepsilon \frac{dE}{dt}$ signifies that a variable electric field $(\frac{dE}{dt})$ creates, similar to a current (J) a variable magnetic field H [3, 8].

Remarks:

1) Maxwell's equations are valid in all three regimes.

2) To obtain the equations in RS, it suffices to set : $\frac{d}{dt} = 0$.

3) To obtain the equations in RQS, it suffices to set : $\frac{d\rho}{dt} \approx 0$ & $\frac{dE}{dt} \approx 0$.

4) MA and MF equations show that the electric field E and the magnetic field H are interconnected, creating what is referred to as the « electromagnetic field ».

IV.3.2. Solved explanatory example (09)

Determine the electric field produced by a point charge q in the surrounding space using Maxwell's equations.

Solution :

Starting from Maxwell's-Gauss's equation: $\oint E.dS = \frac{q}{\varepsilon_0} \Rightarrow \varepsilon_0.E \oint dS = q \Rightarrow \varepsilon_0.E.4\pi r^2 = q$

$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$, which is the same formula given in chapter I by equation (I.4).

IV.4. Localized Ohm's Law

The localized Ohm's Law is the expression of current density J in a conductor in terms of the applied electric field E. As illustrated in the figure below, consider a conductor of length l, cross-sectional area S, and resistance R, subjected to a voltage U and carrying a current I [3].

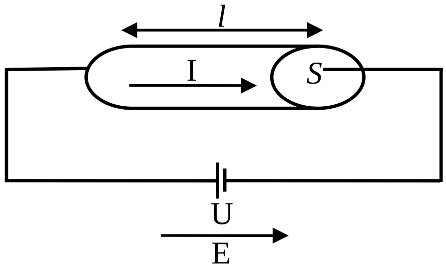


Fig.IV.2

The generalized Ohm's Law is: U=RI. Where : $U = \int Edl$; $R = \int \frac{\rho}{S} dl$ and $I = \int JdS$. Where : ρ is the electrical resistivity of the conductor in [Ωm].

By substituting these three integrals into the generalized Ohm's Law, we obtain:

$\int Edl = \int \frac{\rho}{S} dl \int JdS \Leftrightarrow E = \rho J \Rightarrow J = \frac{1}{\rho} E$; Where $\frac{1}{\rho} = \sigma$ is the electrical conductivity of the conductor in [$\Omega^{-1}m^{-1}$]. Thus, we obtain the formula for the localized Ohm's Law [8]:

$$\boxed{J = \sigma E \; [A/m^2]} \quad \text{(IV.5)}$$

IV.5. Boundary conditions

Consider two points, P_1 and P_2, infinitesimally close to point P located on an interface (a fictitious boundary of separation) between two different dielectric media.

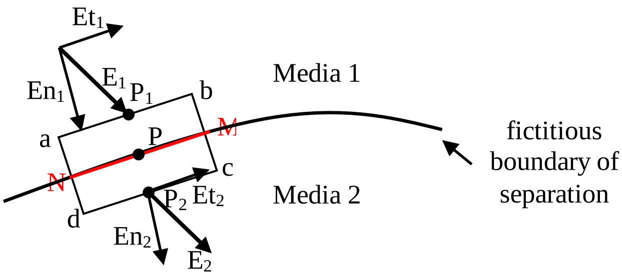


Fig.IV.3.

In this section, we aim to determine the transition conditions (boundary conditions) of an electromagnetic field (electric field and magnetic field) from a dielectric media to another.

IV.5.1. Electric field

The electric field E can be decomposed into tangential component E_t and normal (perpendicular) component E_n with respect to the interface of separation, thus: $E=E_t+E_n$.

a) Tangential components

When taking the integral form of the MF equation $\oint E.dl = -\frac{\partial}{\partial t}\int B.dS$, the closed contour considered in the above figure (IV.3) is the rectangle (abcd) located on either side of the separation interface, thus:

$$\oint E.dl = \int_{abcda} E.dl = E_{n1}.aN + E_{n2}.Nd + E_{t2}.dc + E_{n2}.cM + E_{n1}.Mb + E_{t1}.ba$$

While the two points P_1 and P_2 are infinitely close to the point P located on the separation interface, we can assume: $aN=Nd=bM=Mc\approx 0$. We then obtain:

$\int_{abcda} E.dl = E_{t2}.dc + E_{t1}.ba$ And knowing that: $ba=-dc$ (assuming the sense of dc is positive), therefore: $\int_{abcda} E.dl = dc(E_{t2} - E_{t1})$.

On the other hand, because $bc \approx 0$, the surface of the rectangle $S = abxbc \approx 0$. Therefore: $-\frac{\partial}{\partial t}\int B.dS \approx 0$. Hence: $\int_{abcda} E.dl = dc(E_{t2} - E_{t1}) = 0$. This implies the relation:

$$\boxed{E_{t2} = E_{t1}} \quad \text{(IV.6)}$$

> ***Conclusion :*** The tangential component of the electric field in medium 1 is equal to the tangential component of the electric field in medium 2.

b) Normal components

Assuming now that the separation interface has a surface charge density ρ_s, considering a cube of thickness (e) as a closed surface (see Fig.IV.4).

By using the integral form of MG, we obtain:

$\oint E.dS = \frac{q}{\varepsilon} \Longrightarrow \oint \varepsilon\, E.dS = q \Longrightarrow \varepsilon \oint E.dS =$

$\varepsilon_1 \int E_{n1}.dS_1 + \varepsilon_2 \int E_{n2}.dS_2 + \varepsilon_{1/2} \int E_{n3}.dS_3$

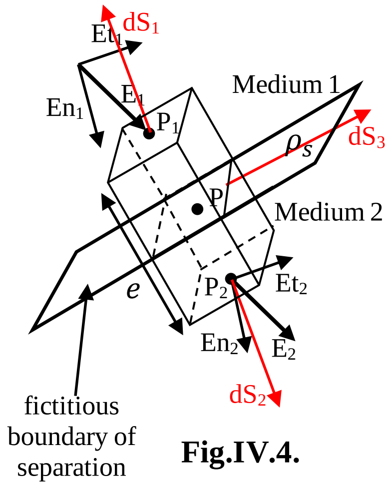


Fig.IV.4.

While the two points P1 and P2 are infinitely close to the point P located on the separation interface, we can assume: $e \approx 0 \Longrightarrow S_3 \approx 0$. Therefore:

$\varepsilon \oint E.dS = \varepsilon_1 \int E_{n1}.dS_1 + \varepsilon_2 \int E_{n2}.dS_2 = \varepsilon_2 E_{n2}.S_2 - \varepsilon_1 E_{n1}.S_1$ (Assuming the sense of dS_2 is positive).

Since $S_1 = S_2 = S$, and for the general case where the separation surface carries a charge $q = \rho_s S$:

$\varepsilon \oint E.dS = S(\varepsilon_2 E_{n2} - \varepsilon_1 E_{n1}) = \rho_s S$, thus: $\varepsilon_2 E_{n2} - \varepsilon_1 E_{n1} = \rho_s$

Therefore, for the general case:

$$E_{n2} = \frac{\rho_s + \varepsilon_1 E_{n1}}{\varepsilon_2} \qquad \text{(IV.7)}$$

For the case of no charge in the interface (the most common case) ($\rho_s = 0$), we then obtain:

$$E_{n2} = \frac{\varepsilon_1}{\varepsilon_2} E_{n1} \qquad \text{(IV.8)}$$

Conclusion : The normal component of the electric field in medium 2 is proportional to the normal component of the electric field in medium 1, depending on the permittivity constants of the two mediums, as well as the surface charge density of their separation interface.

IV.5.2. Magnetic field

Just like the electric field, the magnetic field H can be decomposed into tangential component H_t and normal (perpendicular) component H_n with respect to the separation interface, thus: $H = H_t + H_n$.

a) Tangential components

We follow the same demonstration strategy as in section (IV.5.1.a), but taking into account in this case the presence of the magnetic field H instead of the electric field E (see Fig.IV.5).

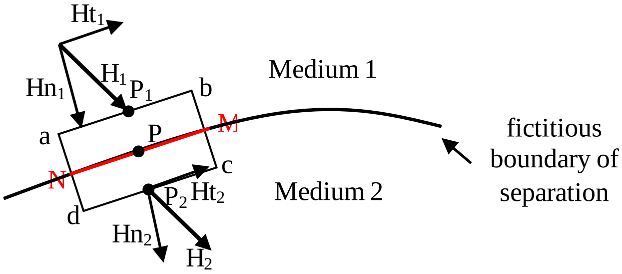


Fig.IV.5.

The application of the integral form of the MA equation ($\oint H.dl = \oint J.dS + \oint \varepsilon \frac{dE}{dt}.dS$) yields:

$\oint H.dl = \int_{abcda} H.dl = H_{n1}.aN + H_{n2}.Nd + H_{t2}.dc + H_{n2}.cM + H_{n1}.Mb + H_{t1}.ba$

While the two points P_1 and P_2 are infinitely close to the point P located on the separation interface, we can assume: *aN=Nd=bM=Mc≈0*. We then obtain:

$\int_{abcda} H.dl = H_{t2}.dc + H_{t1}.ba$; And knowing that: *ba=-dc* (assuming the sense of *dc* is positive), therefore: $\int_{abcda} H.dl = dc(H_{t2} - H_{t1})$.

On the other hand, because $bc \approx 0$, the surface of the rectangle $S = abxbc \approx 0 \Longrightarrow \varepsilon \frac{dE}{dt}.dS = 0$. Now, we are left with the calculation of $\int J.dS$.

Using the right-hand rule, we determine the sense of the current *J* (see Fig.IV.6).

According to the figure, the current *J* flows in the plane (f) perpendicular to the plane formed by the rectangle (abcd), or in other words, perpendicular to the line NM (fictitious separation boundary).

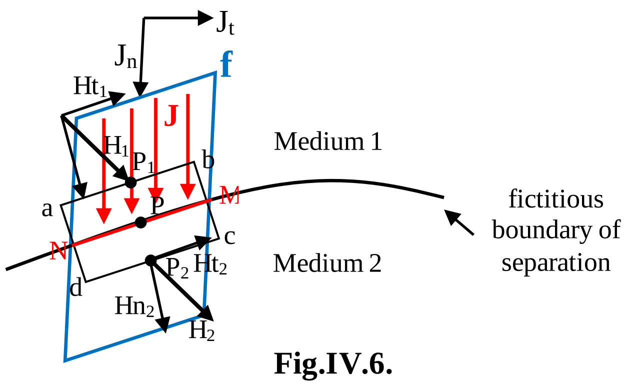


Fig.IV.6.

Therefore, we deduce that the current *J* forms a current sheet (the surface *dS* in the term ($\oint J.dS$) is different from the surface *dS* in the term ($\oint \varepsilon \frac{dE}{dt}.dS$)).

Clarification :

Volumetric Current: The current *I* flows through a volumetric conductor with a cross-sectional area *S* (Fig.IV.7.(a) and (b)). The current density in this case is given by: $J_S = \frac{I}{S}$ which represents a surface current density [3].

Current Sheet: The current *I* flows through a sheet (plane) with a width L (Fig.IV.7.(c) and (d)). The current density in this case is: $J_l = \frac{I}{L}$ representing a linear current density [3].

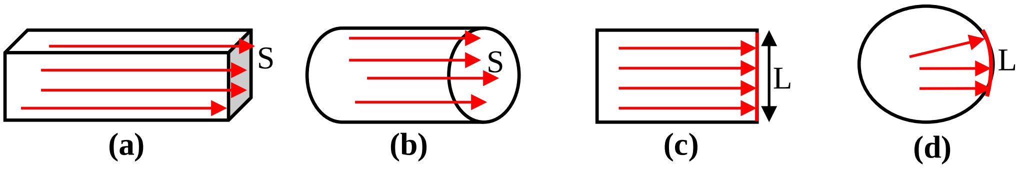


Fig.IV.7. (a), (b) Volumetric Current ; **(c), (d)** Current Sheet.

Therefore, the current passing through the frame is: $I = J_l.L = J_n.NM = J_n.dc$, so:

$\int_{abcda} H.dl = \cancel{dc}(H_{t2} - H_{t1}) = J_n.\cancel{dc} \Longrightarrow (H_{t2} - H_{t1}) = J_n$, in the end, we arrive at:

$$\boxed{H_{t2} = H_{t1} + J_n} \qquad \text{(IV.9)}$$

Conclusion : The tangential component of the magnetic field in medium 2 is proportional to the tangential component of the magnetic field in medium 1, as well as the normal component of the current sheet density J.

b) Normal components

We follow the same demonstration strategy as in section (IV.5.1.b), but taking into account in this case the presence of the magnetic field H instead of the electric field E (see Fig.IV.8).

The application of the integral form of the $M\Phi$ equation ($\oint B.dS = 0$) yields :

$\oint B.dS = \mu \oint H.dS = \mu_1 \int H_{n1}.dS_1 + \mu_2 \int H_{n2}.dS_2 + \mu_3 \int H_{n3}.dS_3 = 0$

At the boundary between the two mediums (boundary conditions): $e \approx 0$, so $S_3 \approx 0$. We then obtain:

$\mu \oint H.dS = \mu_1 \int H_{n1}.dS_1 + \mu_2 \int H_{n2}.dS_2 = \mu_2 H_{n2}.S_2 - \mu_1 H_{n1}.S_1 = 0$ (Assuming the sense of dS_2 is positive).

Since $S_1 = S_2 = S$, we can write:

$S(\mu_2 H_{n2} - \mu_1 H_{n1}) = 0 \Longrightarrow \mu_2 H_{n2} = \mu_1 H_{n1} = 0$, Therefore:

$$\boxed{H_{n2} = H_{n1}\frac{\mu_1}{\mu_2}} \qquad \text{(IV.10)}$$

Or :

$$\boxed{B_{n2} = B_{n1}} \qquad \text{(IV.11)}$$

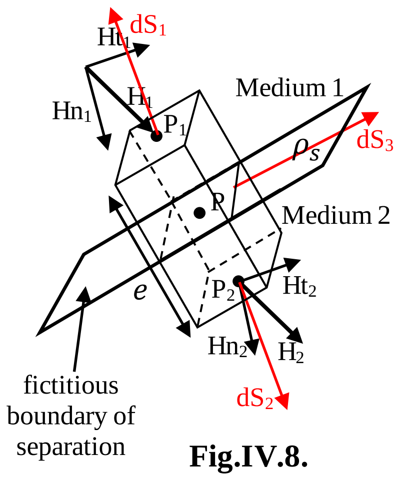


Fig.IV.8.

Conclusion : The normal component of the magnetic field in medium 2 is proportional to the normal component of the magnetic field in medium 1 based on the magnetic permeability constants of the two mediums. Alternatively, the normal component of the magnetic induction in medium 2 is equal to the normal component of the magnetic induction in medium 1.

Chapter V : Propagation of the electromagnetic field

V.1. Mathematical description of propagation

Let $\xi = f(x)$ be a function represented in the figure (Fig. 1) by a solid curve. When this function propagates in the positive direction of the x-axis, at a distance $x=x_0$, we obtain the function $\xi = f(x+x_0)$. Similarly, for propagation in the negative direction, we obtain $\xi = f(x-x_0)$ (see the figure below) [3].

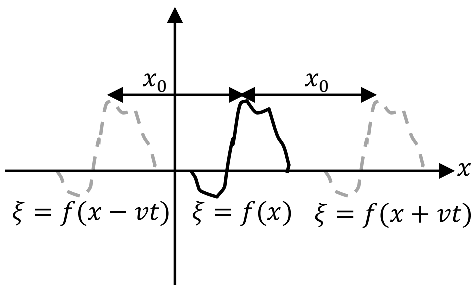


Fig.V.1. Propagation of a function ξ without deformation.

On the other hand, with $x=vt$, where v is the propagation velocity of the curve, and t is the propagation time, we can write: $\xi = f(x+vt)$ for the curve moving to the right, and $\xi = f(x-vt)$ for the curve moving to the left.

<u>Conclusion :</u> A physical phenomenon that propagates without deformation in either the positive or negative direction can be described by a mathematical expression of propagation in the form: $\xi = f(x \pm vt)$.

V.2. Equation of propagation of any wave

<u>Definition:</u> A wave is the propagation of a disturbance causing, in its passage, a reversible variation of the local physical properties of the medium. It travels with a determined velocity that depends on the characteristics of the propagation medium. For example, when whistling, a disturbance is created due to the fluctuation of air pressure between the lips, and it can propagate outward in all directions like a wave. There are three main types of waves: mechanical waves, gravitational waves, and electromagnetic waves.

Considering now any wave (for example, a mechanical wave) created at point "m" on a vibrating string and progressing towards point "p" (in the positive direction of the x-axis) with a propagation velocity v while maintaining the same shape (see the figure below).

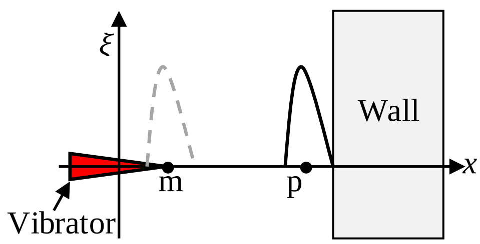


Fig.V.2. Wave experiment on a vibrating string.

The disturbance (wave) at point "p" at time "t" is the same as at the source at point "m" at the previous instant (t – τ), so we can write:

$$\xi_p(t) = \xi_m(t - \tau) \qquad \text{(V.1)}$$

Where:

τ is the time delay or propagation time.

Suppose now that the wave $\xi_p(t)$ propagates back towards the origin "m" in the negative direction of the x-axis with the same previous propagation velocity. Therefore, equation (V.1) becomes:

$$\xi_p(t) = A_m(t+\tau) \qquad \text{(V.2)}$$

Where: $\tau = \frac{x}{v}$;

If we assume propagation towards $x > 0$, we set:

$$\begin{cases} \xi_p(t) = f(t) \\ \xi_m(t-\tau) = f(t-\tau) = f\left(t-\frac{x}{v}\right) = f(u) \end{cases} \qquad \text{where: } u = t - \frac{x}{v}$$

Calculation of the first and second derivatives of ξ with respect to time *t* :

$$\begin{cases} \frac{\partial \xi_m(t-\tau)}{\partial t} = \frac{\partial f(u)}{\partial t} = \frac{\partial f(u)}{\partial u}\frac{\partial u}{\partial t} = f'(u) \ldots\ldots\ldots\ldots\ldots\ldots \text{(first derivative)} \quad \text{(V.3)} \\ \frac{\partial^2 \xi_m(t-\tau)}{\partial t^2} = \frac{\partial^2 f(u)}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f(u)}{\partial t}\right) = \frac{\partial}{\partial t} f'(u) = \frac{\partial f'(u)}{\partial u}\frac{\partial u}{\partial t} = f''(u) .. \text{(second derivative)} \text{(V.4)} \end{cases}$$

Calculation of the first and second derivatives of ξ with respect to *x* :

$$\begin{cases} \frac{\partial \xi_m(t-\tau)}{\partial x} = \frac{\partial f(u)}{\partial x} = \frac{\partial f(u)}{\partial u}\frac{\partial u}{\partial x} = -\frac{1}{v} f'(u) \ldots\ldots\ldots\ldots\ldots \text{(first derivative)} \text{(V.5)} \\ \frac{\partial^2 \xi_m(t-\tau)}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial \xi_m(t-\tau)}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\frac{1}{v} f'(u)\right) = -\frac{1}{v}\frac{\partial f'(u)}{\partial u}\frac{\partial u}{\partial x} = \frac{1}{v^2} f''(u) (2^{nd} \text{ derivative}) \text{(V.6)} \end{cases}$$

By combining equations (V.4) and (V.6), we obtain the propagation equation for the quantity ξ_m along the *x*-axis:

$$\boxed{\frac{\partial^2 \xi_m(t-\tau)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \xi_m(t-\tau)}{\partial t^2}} \qquad \text{(V.7)}$$

So, for propagation in any direction, we write:

$\frac{\partial^2 \xi_m}{\partial x^2} + \frac{\partial^2 \xi_m}{\partial y^2} + \frac{\partial^2 \xi_m}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \xi_m}{\partial t^2}$; Therefore: $\boxed{\nabla^2 \xi_m = \frac{1}{v^2}\frac{\partial^2 \xi_m}{\partial t^2}}$ (V.8)

Generally, the differential propagation equation for the quantity ξ is :

$$\boxed{\nabla^2 \xi = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}} \qquad \text{(V.9)}$$

And along the *x*-axis is: $\boxed{\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}}$ (V.10)

The solution to this equation is of the form: $\boxed{\xi(x,t) = f_1(x-vt) + f_2(x+vt)}$ (V.11)

This solution given by equation (V.11) can then be expressed as the superposition of two waves, one propagating in the positive direction and the other in the opposite direction. For a wave propagating in only one direction, only one of the two functions in equation (V.11) is needed.

V.3. Equation for the propagation of the electromagnetic field in the vacuum

We first recall the differential equations of Maxwell:

$\overrightarrow{div}\,\vec{E} = \frac{\rho}{\varepsilon}$ ….(MG) ; $\overrightarrow{div}\,\vec{B} = 0$….(MΦ) ; $\overrightarrow{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ….(MF); $\overrightarrow{rot}\,H = \vec{j} + \varepsilon\frac{d\vec{E}}{dt}$ ….(MA).

V.3.1. Propagation equation of the electric field $\vec{E}$

Applying the double curl formula to the electric field $\vec{E}$, and using the relevant relationships between operators (see Appendix A) [10], we obtain:

$$\overrightarrow{rot}\left(\overrightarrow{rot}\,\vec{E}\right) = \overrightarrow{grad}\left(\overrightarrow{div}\,\vec{E}\right) - \vec{\Delta}\vec{E}$$

$$= \overrightarrow{grad}\left(\frac{\rho}{\varepsilon_0}\right) - \vec{\Delta}\vec{E} \text{ (According to MG)}$$

$$= \overrightarrow{rot}\left(-\frac{\partial \vec{B}}{\partial t}\right) \text{ (According to MF)}$$

$$= -\left(\frac{\partial(\overrightarrow{rot}\vec{B})}{\partial t}\right) = -\mu_0\frac{\partial \vec{j}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \text{ (According to MA)}$$

Therefore: $\overrightarrow{grad}\left(\frac{\rho}{\varepsilon_0}\right) - \vec{\Delta}\vec{E} = -\mu_0\frac{\partial \vec{j}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$; in a vacuum: *J=0* & $\rho = 0$, so:

$\vec{\Delta}\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$; And according to the vector Laplacian: $\vec{\Delta}\vec{E} = \vec{\nabla}^2\vec{E}$; we then obtain:

$$\vec{\nabla}^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \quad \text{(V.12)}$$

From equation (V.12), and by analogy with the general differential equation for wave propagation in equation (V.9), we deduce that:

$$\mu_0\varepsilon_0 = \frac{1}{v^2} \quad \text{(V.13)}$$

So, equation (V.12) becomes:

$$\vec{\nabla}^2\vec{E} = \frac{1}{v^2}\frac{\partial^2\vec{E}}{\partial t^2} \quad \text{(V.14)}$$

From equation (V.13), we calculate the propagation velocity of the electric field E :

$\frac{1}{v^2} = \mu_0\varepsilon_0 \Longrightarrow v = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{1}{\sqrt{8{,}85.10^{-12}.4\pi10^{-7}}} \approx 3.10^8 m/s = C$ (speed of light) [11].

Finally, we obtain the propagation equation of the electric field $\vec{E}$ [10, 11] :

$$\boxed{\vec{\nabla}^2\vec{E} = \frac{1}{C^2}\frac{\partial^2\vec{E}}{\partial t^2}} \quad \text{(V.15)}$$

Conclusion : The electric field E propagates in a vacuum with the speed of light.

V.3.2. Propagation equation of the magnetic field $\vec{H}$

Applying the double curl formula to the magnetic induction $\vec{B}$, and using the relevant relationships between operators (see Appendix A) [10], we obtain:

$$\overrightarrow{rot}\left(\overrightarrow{rot}\,\vec{B}\right) = \overrightarrow{grad}\left(\overrightarrow{d\iota v}\,\vec{B}\right) - \vec{\Delta}\vec{B}$$

$$= -\vec{\Delta}\vec{B} \text{ (According to M}\Phi : \overrightarrow{d\iota v}\,\vec{B} = 0 \Longrightarrow \overrightarrow{grad}\left(\overrightarrow{d\iota v}\,\vec{B}\right) = 0)$$

On the other hand, we have:

$$\overrightarrow{rot}\left(\overrightarrow{rot}\,\vec{B}\right) = \overrightarrow{rot}(\mu_0\vec{J} + \mu_0\varepsilon_0\frac{d\vec{E}}{dt}) \text{ (According to MA)}$$

Therefore: $$-\vec{\Delta}\vec{B} = \overrightarrow{rot}(\mu_0\vec{J} + \mu_0\varepsilon_0\frac{d\vec{E}}{dt})$$

$$= \mu_0\overrightarrow{rot}(\vec{J}) + \mu_0\varepsilon_0\frac{d(\overrightarrow{rot}(\vec{E}))}{dt}$$

$$= \mu_0\overrightarrow{rot}(\vec{J}) - \mu_0\varepsilon_0\frac{\partial^2(\vec{B})}{dt^2} \text{ (According to MF)}$$

Therefore: $-\vec{\Delta}\vec{B} = \mu_0\overrightarrow{rot}(\vec{J}) - \mu_0\varepsilon_0\frac{\partial^2(\vec{B})}{dt^2}$; in a vacuum: J=0, so :

$\vec{\Delta}\vec{B} = \mu_0\varepsilon_0\frac{\partial^2(\vec{B})}{dt^2}$; And according to the vector Laplacian: $\vec{\Delta}\vec{B} = \vec{\nabla}^2\vec{B}$; we then obtain:

$$\vec{\nabla}^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} \tag{V.16}$$

From equation (V.16), and by analogy with the general differential equation for wave propagation in equation (V.9), and following the same demonstration strategy used in the previous section, we obtain the propagation equation for the magnetic induction $\vec{B}$:

$$\vec{\nabla}^2\vec{B} = \frac{1}{C^2}\frac{\partial^2\vec{B}}{\partial t^2} \tag{V.17}$$

And since $\vec{B} = \mu_0\vec{H}$ (equation (II.3)), we finally obtain the propagation equation for the magnetic field $\vec{H}$ [10] :

$$\boxed{\vec{\nabla}^2\vec{H} = \frac{1}{C^2}\frac{\partial^2\vec{H}}{\partial t^2}} \tag{V.18}$$

<u>Conclusion :</u> The magnetic field H propagates in a vacuum with the speed of light.

V.3.3. Solved explanatory example (10)

Using the Maxwell's equations and with the help of the Lorenz gauge: $\overrightarrow{d\iota v}\,\vec{A} = -\frac{1}{C^2}\frac{\partial V}{\partial t}$; find the propagation equation in a vacuum for the magnetic vector potential $\vec{A}$.

<u>Solution :</u>

According to MA : $\overrightarrow{rot}\,\vec{H} = \vec{J} + \varepsilon_0\frac{d\vec{E}}{dt}$ (in a vacuum $\vec{J} = 0$). Given that $\vec{H} = \frac{1}{\mu_0}\vec{B}$ & $\vec{B} = \overrightarrow{rot}\,\vec{A}$ we have:

$$\frac{1}{\mu_0}\overrightarrow{rot}\left(\overrightarrow{rot}\,\vec{A}\right) = \varepsilon_0\frac{d\vec{E}}{dt} \Leftrightarrow \overrightarrow{rot}\left(\overrightarrow{rot}\,\vec{A}\right) = \varepsilon_0\mu_0\frac{d\vec{E}}{dt} \Leftrightarrow \overrightarrow{rot}\left(\overrightarrow{rot}\,\vec{A}\right) = \frac{1}{C^2}\frac{d\vec{E}}{dt}$$

According to the useful relationships between operators (see Appendix A):

$$\overrightarrow{rot}(\overrightarrow{rot}\,\vec{A}) = \overrightarrow{grad}(d\iota v\,\vec{A}) - \vec{\Delta}\vec{A} = \overrightarrow{grad}(d\iota v\,\vec{A}) - \vec{\nabla}^2\vec{A}$$

Since $\vec{E} = -\overrightarrow{grad}\,\vec{V} - \frac{d\vec{A}}{dt}$ and using the Lorenz gauge:

$$\left(-\frac{1}{c^2}\overrightarrow{grad}\left(\frac{\partial V}{\partial t}\right)\right) - \vec{\nabla}^2\vec{A} = \frac{1}{C^2}\frac{d}{dt}\left(-\overrightarrow{grad}\,\vec{V} - \frac{d\vec{A}}{dt}\right) = \left(-\frac{1}{C^2}\overrightarrow{grad}\left(\frac{\partial V}{\partial t}\right)\right) - \frac{1}{C^2}\frac{\partial^2\vec{A}}{\partial t^2}$$

Finally, we obtain the propagation equation for the magnetic vector potential $\vec{A}$ in a vacuum:

$$\boxed{\vec{\nabla}^2\vec{A} = \frac{1}{C^2}\frac{\partial^2\vec{A}}{\partial t^2}}$$

V.4. Experimental verification

Between the years 1886 and 1888, the German physicist "Heinrich Rudolf Hertz" experimentally verified the existence of electromagnetic waves predicted by "James Clerk Maxwell" in the preceding decade and confirmed their electromagnetic theory published in 1873.

He tirelessly studied the propagation of electromagnetic waves (EMW), which transfer energy from one circuit to another without the need for a conducting wire. He invented and built an oscillator or "exciter" that allowed him to work with very high frequencies.

Hertz's experiment was based on a transmitter consisting of an oscillating LC circuit producing a series of electric arcs and two large spheres connected by a straight conductor of about 3 m, cut in the middle by a spark gap consisting of two small spheres. The spheres are connected to a high-power coil, and the entire setup is isolated from the ground. Charges accumulate in the large spheres until the spark breaks out between the small spheres of the spark gap. The receiver is a loop with ends separated by a small gap. The image in the figure below describes this experiment well.

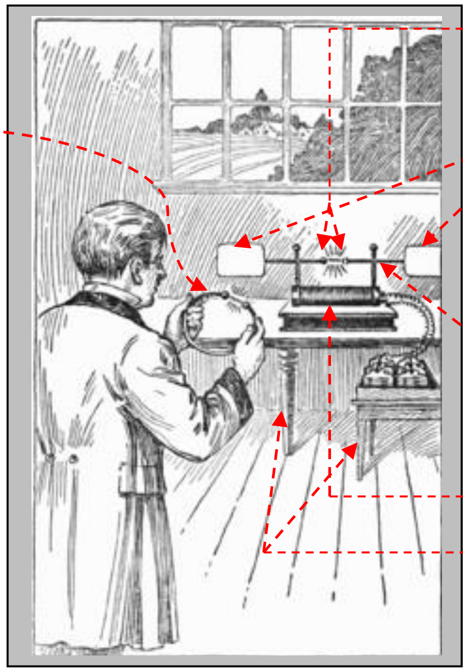


Fig.V.3. Image showing Hertz's experiment for electromagnetic waves.

With each spark (arc) that appears between the two small spheres of the transmitter, a sudden and therefore variable current *i* flows. This current generates an electric field and a magnetic field (electromagnetic field).

With each spark that appears between the two small spheres of the transmitter, Hertz observes that a spark also appears at the terminals of the coil (receiving loop), indicating the presence of a voltage across this coil. In fact, it is an induced electromotive force (emf) caused by the magnetic field *H* created by the transmitter, which propagates to the receiver.

The polarization of the electromagnetic wave (electromagnetic field) is highlighted by the absence of a spark in the receiving loop when it is parallel to the *H* field of the transmitter, meaning the magnetic flux in the loop is zero and cannot induce an emf.

Conclusion : When the electromagnetic field is variable, it becomes a wave that propagates through the air (vacuum).

Consequences of Hertz's experiment

Heinrich Hertz did not witness the application of his device and his discovery, which would enable radio broadcasting and later telecommunications in the 20th century. He contented himself with noting, «**This has no kind of application. It's just an experiment that proves that Master Maxwell was right—we simply have these mysterious**

electromagnetic waves that we cannot see with the naked eye, but they are there ».

The accumulation of information about electromagnetic waves regarding their production, propagation, and absorption has opened the door to the wonderful world of communication as we know it today. In honor of Heinrich Hertz, radio waves are named « hertzian waves ».

V.5. Plane wave

A plane wave is a particular solution of the Maxwell's equations $\xi = f(\omega t - kx)$, which means that at a given instant t, the function ξ takes the same amplitude, phase, and direction at every point with the same coordinate x. This forms a plane represented by E and H (called the wavefront), perpendicular to the x-axis or the propagation direction indicated by the unit propagation vector $\vec{n}$ (see the figure below) [12].

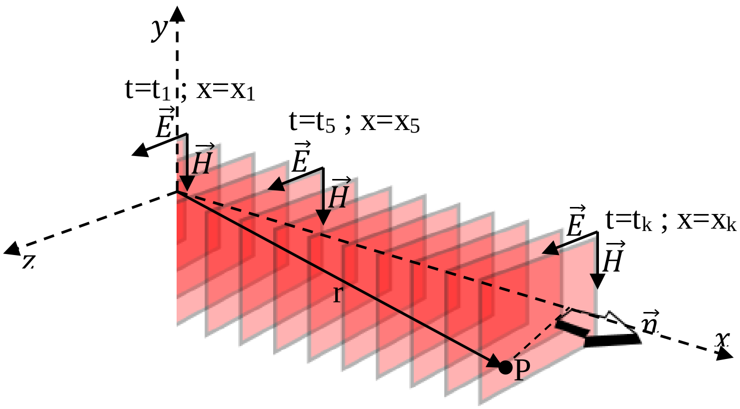


Fig.V.4. Simplified representation of a plane wave.

Let r be the position vector of any point P on a wavefront, we have: $x=n.\,r$, and therefore, we can write:

$$\xi = f\,(\omega t - k\,n.r) \tag{V.19}$$

This form remains valid regardless of the direction of $\vec{n}$: $n = n_x u_x + n_y u_y + n_z u_z$

In the case of a sinusoidal wave propagating in an arbitrary direction $\vec{n}$, we write:

$$\xi = \xi_0 \sin(\omega t - kn.r) \tag{V.20}$$

If the propagation occurs in three-dimensional space, the wave equation must be modified accordingly. It then becomes: $\frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2} + \frac{\partial^2\xi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\xi}{\partial t^2}$

For a sinusoidal wave, we obtain:

$$\xi = \xi_0 \sin(\omega t - kn_x\mathrm{x} - \mathrm{k}n_y\mathrm{y} - \mathrm{k}n_z\mathrm{z}) \tag{V.21}$$

Where: $k^2 = k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{v^2}$

Note: There are several types of wave propagation [3]:

- **Plane waves :** propagate in a single direction (Figure (V.4)).
- **Cylindrical waves:** propagate perpendicular to the axis of a cylinder (Figure (V.5.(a))).
- **Circular waves:** propagate in all directions within a plane (Figure (V.5.(b))). Example: ripples created by a stone thrown into water.
- **Spherical waves:** propagate in all directions (Figure (V.5.(c))).

The circular wave that propagates on a plane is two-dimensional and requires only two spatial coordinates. Therefore, the equation for this wave is:

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}$$

(a) **(b)** **(c)**

Fig.V.5. (a) Cylindrical waves ; **(b)** Circular waves ; **(c)** Spherical waves.

V.6. Characteristics of plane waves

In this section, we assume that the direction of propagation of plane electromagnetic waves is along the positive direction of the x-axis. These plane waves are characterized by:

V.6.1. Transverse wave

According to MG equation: $\overrightarrow{div}\,\vec{E} = \frac{\rho}{\varepsilon}$, but in general, the media through which electromagnetic waves propagate are electrically neutral (such as air). Therefore, $\rho = 0$,and in this case: $\overrightarrow{div}\,\vec{E} = 0$.

Furthermore, by the definition of divergence, we have: $\overrightarrow{div}\,\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$

We refer to our wave in a rectangular coordinate system *xyz*, taking the *x*-axis as the direction along which the wave propagates (Fig. V.4). Therefore:

$$\frac{\partial E_x}{\partial x} = 0 \Longrightarrow E_x(x) = Cst \qquad (V.22)$$

On the other hand, let's recall that the propagation equation for $\vec{E}$ (equation (V.15)) is :

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

We can write: $E = E_x\ u_x + E_y\ u_y + E_z\ u_z$; and as long as the propagation is only along u_x, we obtain:

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \frac{\partial^2 E_x}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 E_x}{\partial t^2} = 0 \Longrightarrow \frac{\partial^2 E_x}{\partial t^2} = 0 \qquad \text{(V.23)}$$

This last equation has two solutions: $\begin{cases} E_x(t) = at + b \\ E_x(t) = Cst \end{cases}$ (V.24)

The first solution is mathematically valid (the first derivative is at and the second derivative is 0), but physically impossible. According to this equation, the electric field increases indefinitely with time without any cause, which is not physically realistic.

The second solution is valid both mathematically (the first derivative is 0, and the second derivative is 0) and physically, as an electric field that remains constant over time is physically plausible.

Therefore, considering equation (V.22) and the second solution to equation (V.24), we can write:

$$E_x(x,t) = Cst \qquad \text{(V.25)}$$

In waves propagation, constant (static) quantities are neglected and set equal to zero, so:

$$E_x(x,t) = 0 \qquad \text{(V.26)}$$

On the other hand, by using the (MΦ) equation and following the same steps of the demonstration, we can also show that:

$$H_x(x,t) = 0 \qquad \text{(V.27)}$$

Conclusion **:** The component of the electromagnetic field along the direction of propagation (E_x, H_x) being zero, the plane wave is located in the yoz plane and therefore perpendicular to the propagation direction ox: the plane wave is **transverse**.
We can define, therefore, that a transverse electromagnetic plane wave is a wave for which the electric and magnetic field vectors lie in a transverse plane or perpendicular to the axis of propagation.

Note: There are other types such as transverse spherical and cylindrical electromagnetic waves.

V.6.2. Characteristic impedance

Let's define: $E = f\left(t - \frac{x}{v}\right); H = g\left(t - \frac{x}{v}\right)$

equation of MF is : $rot\,E = -\mu_0 \frac{\partial H}{\partial t}$

On the other hand, we also have: $rot\,E = \begin{vmatrix} u_x & u_y & u_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & E_y & E_z \end{vmatrix} = -u_y \frac{\partial \mathrm{E_z}}{\partial \mathrm{x}} + u_z \frac{\partial \mathrm{E_y}}{\partial \mathrm{x}}$ (The components of the vector $\vec{E}$ are E_y and E_z, and the propagation of $\vec{E}$ is along the *x*-coordinate, so the derivative is with respect to *x* only $\frac{\partial}{\partial x}$).

On the other hand: $-\mu_0 \frac{\partial H}{\partial t} = -\mu_0 \frac{\partial H_y}{\partial t} u_y - \mu_0 \frac{\partial H_z}{\partial t} u_z$; We then obtain by analogy:

$$\frac{\partial \mathrm{E_z}}{\partial \mathrm{x}} = \mu_0 \frac{\partial H_y}{\partial t} \text{ ………(a)} \qquad \& \qquad \frac{\partial \mathrm{E_y}}{\partial \mathrm{x}} = \mu_0 \frac{\partial H_z}{\partial t} \text{ ………(b)}$$

Furthermore, the equation for MA is: $rot\,H = \varepsilon_0 \frac{dE}{dt}$

We also have: $rot\,H = \begin{vmatrix} u_x & u_y & u_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & H_y & H_z \end{vmatrix} = -u_y \frac{\partial H_z}{\partial x} + u_z \frac{\partial H_y}{\partial x}$ (The same as before with the vector $\vec{E}$).

Where: $\varepsilon_0 \frac{dE}{dt} = \varepsilon_0 \frac{dE_y}{dt} u_y + \varepsilon_0 \frac{dE_z}{dt} u_z$, so by analogy:

$$\frac{\partial H_z}{\partial x} = -\varepsilon_0 \frac{dE_y}{dt} \text{…….(c)} \qquad \& \qquad \frac{\partial H_y}{\partial x} = \varepsilon_0 \frac{dE_z}{dt} \text{……..(d)}$$

If we set: $E_y = f_1(t - \frac{x}{v})$, equation (c) yields: $\frac{\partial E_y}{\partial t} = \frac{\partial f_1(u)}{\partial t} = \frac{\partial f_1(u)}{\partial u} \frac{\partial u}{\partial t} = f_1'(u) = -\frac{1}{\varepsilon_0} \frac{\partial H_z}{\partial x}$

Therefore, the integral of (c) gives: $H_z = \int -\varepsilon_0\, f_1'\,(u) dx$

We have: $\frac{du}{dx} = -\frac{1}{v} \Longrightarrow dx = -v.du$; Substituting, we get:

$$H_z = -\varepsilon_0(-v) \int f_1'(u) du = \varepsilon_0 v f_1(u) + Cte = \varepsilon_0 v f_1(u) \text{ (neglecting the constant).}$$

So, we obtain:

$$H_z = \varepsilon_0 v E_y \quad \text{and} \quad \frac{E_y}{H_z} = \frac{E_y}{\varepsilon_0 v E_y} = \frac{1}{\varepsilon_0 v} = \frac{\sqrt{\varepsilon_0\,\mu_0}}{\varepsilon_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0$$

Therefore:

$$\frac{E_y}{H_z} = Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad \text{(V.28)}$$

* If we set: $E_z = f_2(t - \frac{x}{v})$, equation (d) yields: $\frac{\partial E_z}{\partial t} = \frac{\partial f_2(u)}{\partial t} = \frac{\partial f_2(u)}{\partial u} \frac{\partial u}{\partial t} = f_2'(u) = \frac{1}{\varepsilon_0} \frac{\partial H_y}{\partial x}$

$H_y = \int \varepsilon_0\, f_2'\,(u) dx = -\varepsilon_0 v \int f_2'(u) du = -\varepsilon_0 v f_2(u) + Cte = \varepsilon_0 v f_2(u)$

So, therefore: $H_y = -\varepsilon_0 v E_z$

We now calculate the ratio: $\frac{E_z}{H_y} = \frac{E_z}{-\varepsilon_0 v E_z} = -\frac{1}{\varepsilon_0 v} = -\frac{\sqrt{\varepsilon_0\,\mu_0}}{\varepsilon_0} = -\sqrt{\frac{\mu_0}{\varepsilon_0}} = -Z_0$

So:

$$\frac{E_z}{H_y} = -Z_0 \qquad \text{(V.29)}$$

From equations (V.28) and (V.29), the ratio between the E and H modules is then given by:

$\frac{E}{H}=\frac{\sqrt{E_y^2+E_z^2}}{\sqrt{H_y^2+H_z^2}}=\frac{\sqrt{E_y^2+E_z^2}}{\sqrt{(\frac{-E_z}{Z_0})^2+(\frac{E_y}{Z_0})^2}}=\frac{\sqrt{E_y^2+E_z^2}}{\sqrt{\frac{1}{Z_0{}^2}(E_y^2+E_z^2)}}=\frac{1}{1/Z_0}=Z_0$. Therefore:

$$\boxed{\frac{E}{H}=Z_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}} \qquad \text{(V.30)}$$

The unit of Z_0 is : $[Z_0]=\frac{[E]}{[H]}=\frac{V/m}{A/m}=\frac{V}{A}=\Omega$

Z_0 is called the « characteristic impedance » of the medium in which wave propagation occurs [3]. For a vacuum (air) :

$Z_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=\sqrt{\frac{4\pi 10^{-7}}{8{,}85.10^{-12}}}=120\pi\approx 376{,}81\ \Omega$ (Impedance of air, the medium through which the wave propagates) [3, 11].

V.6.3. Perpendicularity of $\vec{E}$ and $\vec{H}$

As long as we consider propagation along the positive *x*-axis, so :

$E_x=0$ and $H_x=0$ then : $E=E_y\,u_y+E_z\,u_z$ and $H=H_y\,u_y+H_z\,u_z$

The law of perpendicularity : $E.H=0$

Verifying this law: $E.H=E_yH_y+E_zH_z=E_y\left(\frac{E_z}{-Z_0}\right)+E_z\left(\frac{E_y}{Z_0}\right)=0\Longrightarrow E\perp H$

V.6.4. Propagation direction

We calculate the cross product of $\vec{E}$ and $\vec{H}$:

$\vec{E}\wedge\vec{H}=\begin{vmatrix} u_x & u_y & u_z \\ 0 & E_y & E_y \\ 0 & H_y & H_z \end{vmatrix}=u_x\left(E_yH_z-E_zH_y\right)=u_x\left(Z_0H_z.H_z-Z_0H_y.H_y\right)=Z_0\left(H_y^2+\right.$ $\left.H_z^2\right)u_x=Z_0H^2u_x$; And, since $u_x=n$, we can write:

$$\boxed{E\wedge H=Z_0H^2n} \qquad \text{(V.31)}$$

Therefore, the cross product $\vec{E}\wedge\vec{H}$ determines the direction of propagation of the electromagnetic field (electromagnetic wave) [3].

V.7. Propagation in an arbitrary direction

We consider the following progressive wave function that propagates in the positive direction of the x-axis in the (xoy) plane:

$$f(x,t)=A\sin(\omega t-kx+\varphi) \qquad \text{(V.32)}$$

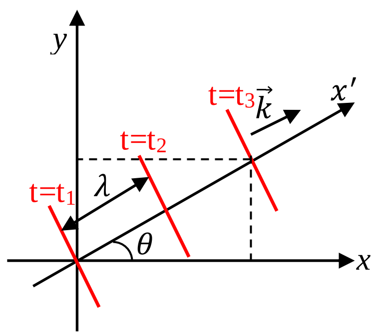


Fig.V.6. Propagation of a plane wave in any direction.

If this plane wave propagates in a given arbitrary direction (in the figure here, the direction is along x'), the previous function becomes:

$$f(x',t) = Asin(\omega t - kx' + \varphi) \quad \text{(V.33)}$$

Where: $x' = x\cos\theta + y\sin\theta$

Therefore, we obtain:

$$f(x,y,t) = Asin(\omega t - kx\cos\theta - ky\sin\theta + \varphi) \quad \text{(V.34)}$$

The propagation vector $\vec{k}$ whose norm is defined by $\vec{k} = \frac{2\pi}{\lambda}$ (λ is the wavelength), can be written as follows:

$$\vec{k} = k_x\cos\theta\,\vec{\imath} + k_y\sin\theta\,\vec{\jmath}$$

Then the wave function becomes:

$$f(x,y,t) = Asin(\omega t - k_x x\cos^2\theta - k_y y\sin^2\theta + \varphi) \quad \text{(V.35)}$$

In a generalized manner, for a wave propagating in space, the propagation vector thus has 3 components: k_x, k_y et k_z.

V.8. Velocity and wavelength

V.8.1. Wave velocity

A wave is a disturbance that travels through a medium. It is possible to associate two wave velocities with it, namely the phase velocity and the group velocity, which, at times, are not equal:

a) Phase velocity

The phase velocity of a wave is the speed at which the phase of the wave propagates through space. By selecting a specific point on the wave (such as the crest), this immaterial point moves through space at the phase velocity.

Consider a plane wave defined in space and time by: $\xi(x,t) = \xi_0\cos(\omega t - kx + \varphi_0)$. The wavefront is formed by all points with the same value of ξ, and consequently the same value of the phase φ, Therefore, the magnitudes of *E* and *H* of the wave, as well as the phase φ are constant. We can thus write:

$\varphi = \omega t - kx + \varphi_0 = Cte$; The derivative is therefore:

$$d\varphi = d(\omega t - kx + \varphi_0) = d(Cte) \Longrightarrow \omega dt - kdx = 0 \Longrightarrow \omega dt = kdx$$

$$\Longrightarrow \frac{dx}{dt} = \boxed{v = \frac{\omega}{k}} \quad \text{(V.36)}$$

Where *v* here is referred to as « phase velocity » [11].

b) Group velocity

The group velocity is defined as the speed at which a signal composed of a narrow band or a group of frequency components propagates. More rigorously, the group velocity is defined as the derivative of angular frequency with respect to the wave number (magnitude of the wave vector $\vec{k}$) associated with that angular frequency:

$$v_g = \frac{\partial \omega}{\partial k} \tag{V.37}$$

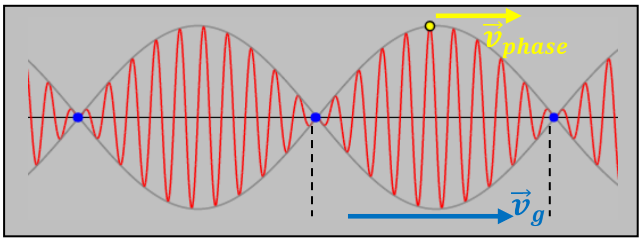


Fig.V.7. Phase and group velocities of a superposition of two waves.

V.8.2. Wavelength

The wavelength is defined as the distance between two consecutive maxima of the amplitude or alternatively as the distance traveled by the wave during one period T of oscillation (see Figure (V.8)).

Let : $\varphi = \omega t - kx$

The phase over time is : $\varphi(t) = \omega t$; For a period T, i.e., for t=T :

$$\omega T = 2\pi \Longrightarrow \quad T = \frac{2\pi}{\omega} \tag{V.38}$$

The phase in space is : $\varphi(x) = kx$; For a period in space or wavelength λ, i.e., for $x = \lambda$:

$$-k\lambda = 2\pi \Leftrightarrow k\lambda = -2\pi \Leftrightarrow k\lambda = 2\pi \Longrightarrow \quad \lambda = \frac{2\pi}{k} \tag{V.39}$$

By combining equations (V.38) and (V.39), we obtain:

$\omega T = k\lambda$ from which $\frac{\omega}{k} = \frac{\lambda}{T}$. Since $\frac{\omega}{k} = v$ and $\frac{1}{T} = f$ (wave frequency), we deduce the relationship:

$$v = \lambda f \tag{V.40}$$

<u>Clarification:</u> The period is the temporal equivalent of the wavelength; it is the minimum time elapsed between two identical repetitions of the wave at the same point. For a sinusoidal wave, the wavelength is the distance between two successive peaks of the same sign.

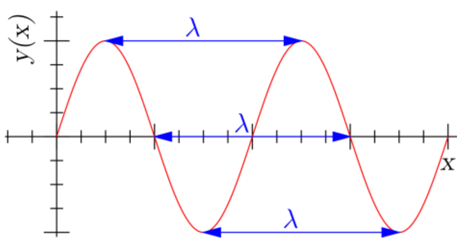


Fig.V.8. Wavelength.

V.9. Propagation of electromagnetic energy

V.9.1. Propagated electromagnetic energy and Poynting vector

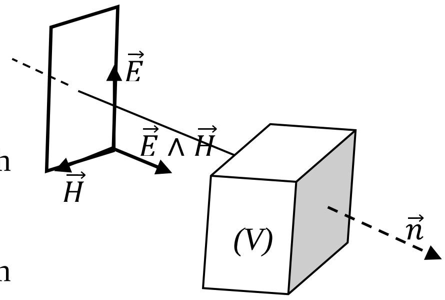


Fig.V.9. Propagation of electromagnetic energy.

Electromagnetic waves carry electromagnetic power with them through space to distant receiving points.

Consider an incoming plane wave in a volume V as shown in the figure below.

According to the relevant relationships between operators:

$$\overrightarrow{div}\left(\vec{E} \wedge \vec{H}\right) = \vec{H}\overrightarrow{rot}\,\vec{E} - \vec{E}\overrightarrow{rot}\,\vec{H} \qquad \text{(V.41)}$$

Substituting the equations of (MF): $\overrightarrow{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and of (MA) : $\overrightarrow{rot}\,H = \vec{j} + \varepsilon\frac{d\vec{E}}{dt}$ (in vacuum: $\vec{j} = 0$ and $\varepsilon = \varepsilon_0$) into equation (V.41) yields:

$$\overrightarrow{div}\left(\vec{E} \wedge \vec{H}\right) = -\vec{H}\frac{\partial \vec{B}}{\partial t} - \vec{E}\varepsilon_0\frac{d\vec{E}}{dt} \qquad \text{(V.42)}$$

Where :

$$\vec{H}\frac{\partial \vec{B}}{\partial t} = \vec{H}\frac{\partial(\mu_0\vec{H})}{\partial t} = \frac{1}{2}\frac{\partial(\mu_0\vec{H}\vec{H})}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu_0\vec{H}^2\right) = \frac{\partial}{\partial t}(w_m) \qquad \text{(V.43)}$$

$$\vec{E}\varepsilon_0\frac{d\vec{E}}{dt} = \frac{1}{2}\frac{\partial(\varepsilon_0\vec{E}\vec{E})}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0\vec{E}^2\right) = \frac{\partial}{\partial t}(w_e) \qquad \text{(V.44)}$$

Equation (V.42) can therefore be written as follows:

$$\overrightarrow{div}\left(\vec{E} \wedge \vec{H}\right) = -\frac{\partial}{\partial t}(w_e + w_m) \qquad \text{(V.45)}$$

By integrating both sides of this equation over the relevant volume, we obtain:

$$\int -\overrightarrow{div}\left(\vec{E} \wedge \vec{H}\right)dv = \frac{\partial}{\partial t}\int(w_e + w_m)dv \qquad \text{(V.46)}$$

By using the Green-Ostogradsky theorem (or the divergence theorem), we obtain:

$$\oint -\left(\vec{E} \wedge \vec{H}\right)ds = \frac{\partial}{\partial t}\int(w_e + w_m)dv \qquad \text{(V.47)}$$

The term : $\vec{E} \wedge \vec{H} = \vec{\wp}$ is called the « Poynting vector » and represents the energy flux per unit area, with its unit being [watts/meter^2] [10, 13]. Therefore, the previous equation can be rewritten as follows:

$$\oint -\vec{\wp}ds = \frac{\partial}{\partial t}\int(w_e + w_m)dv \qquad \text{(V.48)}$$

The term $\frac{\partial}{\partial t}(w_e + w_m)$ represents the increase in electromagnetic energy in volume V [13]. And the term $\int(E \wedge H).ds$ represents the outgoing energy from V.

Conclusion: Equation (V.48) indicates that the total power of a plane wave flowing through a closed surface at any given time is equal to the sum of the rates of increase of electric and magnetic energies stored within the closed volume.

V.9.2. Solved explanatory example (11)

Find the expression for the Poynting vector on the surface of a straight conductor wire with radius r and conductivity σ, carrying a direct current I.

Solution :

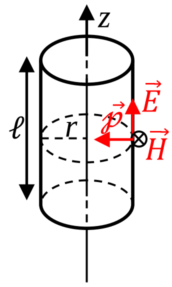


Fig.V.10.

Direct current $\Longrightarrow$ the current in the wire is uniformly distributed across its cross-sectional area. Let's assume that the axis of the wire coincides with the z-axis. The figure below shows a segment of length l of the straight wire.

The Poynting vector is : $\vec{\wp} = \vec{E} \wedge \vec{H}$; so we need to find $\vec{E}$ and $\vec{H}$ first.

We have: $I = \int \vec{J}.ds = \vec{J}\pi r^2 \Rightarrow \vec{J} = \frac{I}{\pi r^2} u_z$

According to Ohm's law localized: $\vec{J} = \sigma\vec{E} \Rightarrow \vec{E} = \frac{\vec{J}}{\sigma} = \frac{I}{\sigma\pi r^2} u_z \ldots\ldots(*)$

Applying Ampère's theorem on the surface of the wire: $\oint \vec{H}.dl = I \Rightarrow \vec{H}.2\pi r = I$

$$\Rightarrow \vec{H} = \frac{I}{2\pi r} u_\phi \ \ldots.(**)$$

From (*) and (**) : $\vec{\wp} = \vec{E} \wedge \vec{H} = \left(\frac{I}{\sigma\pi r^2} u_z\right) \wedge \left(\frac{I}{2\pi r} u_\phi\right) = \frac{I}{\sigma\pi r^2} \cdot \frac{I}{2\pi r} (u_z \wedge u_\phi)$

$$\Rightarrow \vec{\wp} = \frac{I^2}{2\sigma\pi^2 r^3} u_r$$

The flux of $\vec{\wp}$ is entering the cylinder, so :

$$\boxed{\vec{\wp} = -\frac{I^2}{2\sigma\pi^2 r^3} u_r}$$

V.10. Reflection and transmission of waves

V.10.1. In a perfect conductor

As previously known in section (I.13.1), the electric field in a perfect conductor is zero; therefore, the wave incident on this conductor is not transmitted through it but is entirely reflected (see Figure (V.11.(a))).

V.10.2. In a perfect dielectric

As previously known in section (I.13.2), and since the electric field passes through a perfect insulator (perfect dielectric), the wave incident on a perfect dielectric splits into a reflected wave and a transmitted wave (see Figure (V.11.(b))).

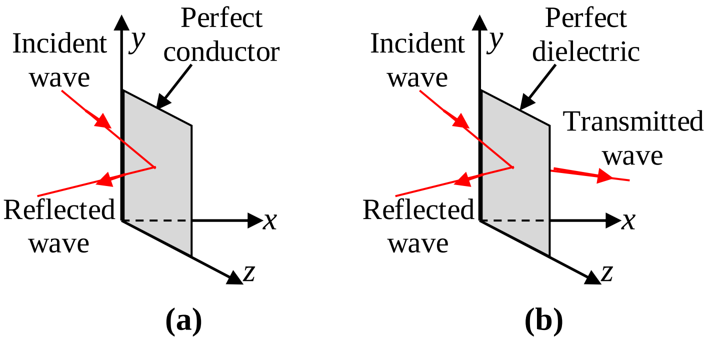


Fig.V.11. Representation of reflection and transmission of an electromagnetic wave.

V.11. Guided waves

In the field of communication, waves that carry information through free space (via satellite, mobile phones, walkie-talkies, etc.) can be deviated or disrupted by material obstacles or atmospheric disturbances. Similarly, in the field of medicine, laser beams used to destroy a cancerous tumor or repair a retina can damage healthy organs. Therefore, in certain domains, it is necessary to control the propagation of waves using devices called « waveguides » [10].

Contrary to free-space propagation, guided propagation involves directing an electromagnetic wave within a space defined by metallic or dielectric interfaces, from a transmitter to a receiver.

There are several types of waveguides:

- **Rectangular waveguide:** The wave entering the guide undergoes a series of multiple reflections before emerging on the other side (Fig.V.12.(a)).
- **Cylindrical waveguide:** Such as the optical fiber, which is a cylindrical metallic tube (Fig.V.12.(b)).
- **Ionospheric propagation guide:** The waves emitted by the radio station are reflected, on one hand, by the Earth's surface and, on the other hand, by the ionospheric atmospheric layer. Satellite TV waves: The satellite reflects the waves emitted by the TV station back to Earth (Fig.V.12.(c)).

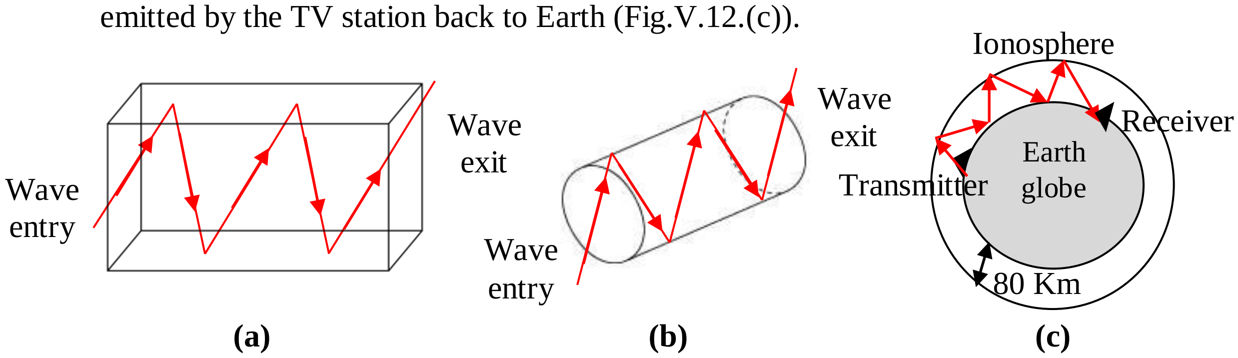


Fig.V.12. Graphical representations of waveguides : **(a)** Rectangular ; **(b)** Cylindrical ; **(c)** Of ionospheric propagation.

V.12. Electromagnetic radiation spectrum

The electromagnetic spectrum refers to the distribution of electromagnetic waves based on their frequency, wavelength, or energy. It theoretically extends from (0) to (∞) in frequency or wavelength in a continuous manner. However, the accessible and utilized part of the spectrum has been divided into several broad classes of radiation, designated by specific names, as illustrated in the table in Annex (C).

V.12.1. Classification of the electromagnetic spectrum

The classification of the electromagnetic spectrum, starting from the most energetic waves to the least energetic ones, is successively as follows:

1) γ rays: $f \in [3.\,10^{18}$- $5.\,10^{22}$ Hz] ; $\lambda \in [10^{-14}$- 10^{-12}m] ; Very energetic : W∈[10^{4}- 10^{7}eV]. They are naturally emitted by radioactive elements. They easily penetrate matter and are highly dangerous to living cells (they can cause burns, cancers, and genetic mutations) [3, 10].

Applications : Cancer treatment, food preservation.

2) X-rays : $f \in [3.10^{17}$- $5.\,10^{19}$Hz] ; $\lambda \in [10^{-12}$- 10^{-8}m] ; Very energetic W∈[1,2. 10^{3}- 2,4. 10^{5}eV]. Penetrating material bodies to varying degrees, X-rays are somewhat less harmful than γ rays.

Applications : Used in medicine for X-rays (the relatively greater absorption by bones compared to tissue allows for precise "photography"), in industry for baggage screening at airports, and in scientific research for studying matter (synchrotron radiation) [10].

3) Ultraviolet rays : $f \in [8.\,10^{14}$- $3.\,10^{17}$Hz] ; $\lambda \in$ [$10^{-8}m$- 390 nm] ; Relatively energetic W∈[3- 2.10^{3}eV]. These waves are generated by hot sources involving high energies, such as the sun. They are harmful to the skin. However, a significant portion of ultraviolet radiation is absorbed by the atmospheric ozone layer.

Applications : Treatment of certain diseases, sterilization of surgical instruments (as some microorganisms that absorb UV radiation are destroyed) [10].

4) Visible spectrum (light) : $f \in [4.10^{14}$- 8.10^{14}Hz] ; $\lambda \in$[390 nm- 780 nm]; W∈[1.6- 3.2 eV]. Corresponds to the very narrow portion of the electromagnetic spectrum perceptible by the human eye. It includes the six colors that make up white light (red, orange, yellow, green, blue, violet).

Applications : Lighting, lasers, photography, etc.

5) Infrared spectrum : f ∈[3.10^{11}- 4.10^{14}Hz] ; λ ∈[780 nm- 1 mm] ; W∈[10^{-3}- 1.6eV]. Infrared radiation is naturally emitted by all bodies with temperatures above absolute zero (0K=-273.15°C= -459.67°F).

Applications : Remote control, satellite communication, remote sensing. Certain spectral bands of infrared are used to measure the temperature of land and ocean surfaces [10].

6) Microwaves : f ∈[10^{9}- 3.10^{11}Hz] ; λ ∈[1mm- 1 m]; W∈[10^{-5}- 10^{-3}eV].

Applications : Microwave oven, satellite communications, mobile phones, radar, geolocation, remote sensing of moving objects [10].

7) Radio and TV waves : f ∈[a few kHz- 10^{9}Hz] ; λ ∈[1m- hundreds of Km] ; They carry little energy: W∈[10^{-5}-0 eV].

Applications : Used for the transmission of information (radio, television).

Note: The electromagnetic spectrum including the three regions - infrared, visible, and ultraviolet - forms what is called the optical spectrum (or optical domain).

V.12.2. Solved explanatory example (12)

Consider an electromagnetic wave propagating in a vacuum at the speed of light, with a wavelength of 600 nm. Specify the electromagnetic spectrum range in which this wave is located.

Solution :

We can specify the electromagnetic spectrum range through the calculation of frequency:

According to equation (V.40), we have: $v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{C}{\lambda} = \frac{3.10^{8}}{600.10^{-9}} = 5.10^{14} Hz$ ∈[4.10^{14}- 8.10^{14}Hz] ⟹ the wave is in the visible spectrum.

Chapter VI : Reflection and transmission of electromagnetic waves

VI.1. Normal incidence of an electromagnetic wave

Consider an electromagnetic wave propagating in medium 1, with the direction of propagation considered normal to the interface between two media, 1 and 2, as shown in the figure (VI.1) below.

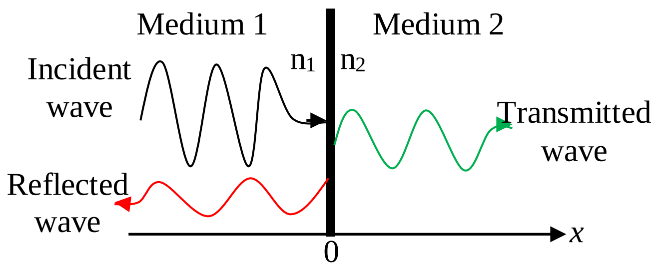


Fig.VI.1. Reflection and transmission of a wave at normal incidence on a flat interface. [14].

According to the figure, the incident wave propagates in medium 1 ($x<0$) along the positive direction of the x-axis and normal to the separating surface located at $x=0$, while the reflected wave propagates in the opposite direction (negative x). On the other hand, the transmitted wave continues its propagation in medium 2 along the positive direction of the x-axis [14].

The directions of the three vectors u_i, u_r, and u_t (incident, reflected, and transmitted) are related as follows:

- The directions of incidence, reflection, and transmission lie in the same plane perpendicular to the separating surface.
- The angle of incidence (θ_i) = the angle of reflection (θ_r) [3].
- $\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_i}{v_r}$ (where v_i and v_r are the velocities of the incident and reflected waves, respectively) [3].

Based on the previous figure, let's assume that the incident wave is expressed as follows:

$$\vec{E} = E \exp i(kx - \omega t) \quad \text{(VI.1)}$$

$\vec{x}$ is the position vector.

at x=0 :

$$\vec{E} = E \exp(-i\omega t) \quad \text{(VI.2)}$$

In medium (1), there are both the incident and reflected waves, while in medium (2), only the transmitted wave exists. Therefore, at the separating interface:

$$\vec{E_i} + \vec{E_r} = \vec{E_t} \quad \text{(VI.3)}$$

For the incident wave:

$$\begin{cases} \overrightarrow{E_i} = E_{i_y} \\ \overrightarrow{B_i} = \frac{\eta_1}{c} E_{i_y} \end{cases} \quad \text{(VI.4)}$$

For the reflected wave:

$$\begin{cases} \overrightarrow{E_r} = E_{r_y} + E_{r_z} \\ \overrightarrow{B_r} = \frac{\eta_1}{c} E_{r_z} - \frac{\eta_1}{c} E_{r_y} \end{cases} \quad \text{(VI.5)}$$

For the transmitted wave:

$$\begin{cases} \overrightarrow{E_t} = E_{t_y} + E_{t_z} \\ \overrightarrow{B_t} = \frac{\eta_2}{c} E_{t_y} - \frac{\eta_2}{c} E_{t_z} \end{cases} \quad \text{(VI.6)}$$

Where : η is the intrinsic impedance of the material in ohms (Ω).

We define the reflection coefficient "Γ" as the ratio of the phasors of the reflected and incident electric fields at the separating interface, expressed as:

$$\boxed{\Gamma = \frac{E_{r_y}}{E_{i_y}} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}} \quad \text{(VI.7)}$$

On the other hand, we also define another coefficient called the « Transmission Coefficient » « τ », given by:

$$\boxed{\tau = \frac{E_{t_y}}{E_{i_y}} = \frac{2\eta_1}{\eta_1 + \eta_2}} \quad \text{(VI.8)}$$

In the case where : $\eta_1 = \eta_2 \Rightarrow \Gamma = 0 \;\&\; \tau = 1 \Rightarrow$ the incident wave is completely transmitted (with no reflected wave).

VI.2. Oblique incidence of an electromagnetic wave

Let's consider an incident plane wave as shown in the figure below. The propagation vector β and the normal to the plane of separation make an angle θ_i. We assume that the conductivity of both materials is zero.

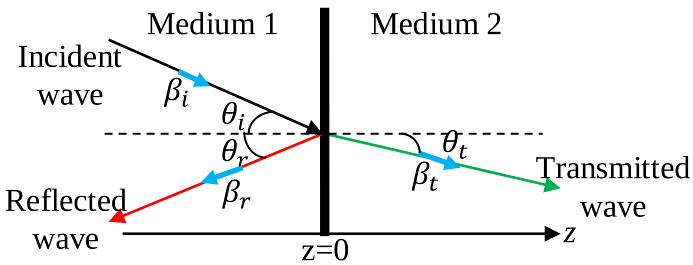


Fig.VI.2. Reflection and transmission of a wave at oblique incidence on a flat interface.

The general equation of « Snell's-Descartes's » is given by:

$$\frac{v_{p1}}{\sin\theta_i} = \frac{v_{p1}}{\sin\theta_r} = \frac{v_{p2}}{\sin\theta_t} \quad \text{(VI.9)}$$

With : v_{p1}, v_{p2} are, respectively, the wave propagation velocities in media 1 and 2.

It is noted that this equation is valid for all types of materials, with or without losses. We define the refractive index n of a material (a medium) as the ratio of the propagation velocity in a vacuum (speed of light) to the velocity in the respective material:

$$n = \frac{c}{v_p} \quad \text{(VI.10)}$$

From these last two equations, it is easily deduced that: θ_i & θ_r are situated in the same medium (air), so the incident and reflected waves have the same propagation velocity: $v_{p1} = c$. Therefore:

$$\frac{v_{p1}}{\sin\theta_i} = \frac{v_{p1}}{\sin\theta_r} = \frac{c}{\sin\theta_i} = \frac{c}{\sin\theta_r} \Rightarrow \sin\theta_i = \sin\theta_r \Rightarrow \theta_i = \theta_r$$

We then obtain the law of reflection:

$$\boxed{\theta_i = \theta_r} \quad \text{(VI.11)}$$

From equation (VI.10): $v_{p1} = \frac{c}{n_1}$ & $v_{p2} = \frac{c}{n_2}$; and from equation (VI.9), we will have:

$$\frac{c}{n_1 \sin\theta_i} = \frac{c}{n_2 \sin\theta_t} \Rightarrow n_1 \sin\theta_i = n_2 \sin\theta_t$$

We then obtain the law of refraction:

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t} \quad \text{(VI.12)}$$

VI.3. Polarization of an obliquely incident plane wave

VI.3.1. Perpendicular polarization

It is said that a plane wave has perpendicular polarization when the vector of the electric field of the incident, reflected, and transmitted waves is oriented perpendicular to the plane of incidence formed by the normal to the separating surface and the wave propagation vectors (see Fig. VI.3) [14].

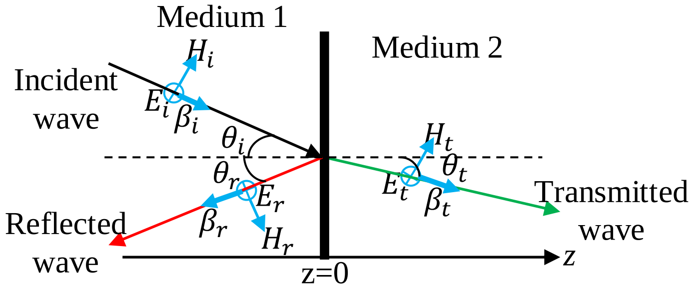


Fig.VI.3. Reflection and transmission of an obliquely incident wave on a flat interface with perpendicular polarization.

At z=0, the boundary conditions are expressed as:

$$\overrightarrow{E_{iy}} + \overrightarrow{E_{ry}} = \overrightarrow{E_{ty}} \quad \text{(VI.13)}$$

$$\overrightarrow{H_{ix}} - \overrightarrow{H_{rx}} = \overrightarrow{H_{tx}} \quad \text{(VI.14)}$$

Expressing both equations in terms of the total fields, we obtain:

$$\overrightarrow{E_i} + \overrightarrow{E_r} = \overrightarrow{E_t} \quad \text{(VI.15)}$$

$$\overrightarrow{H_i} \cos\theta_{i,r} - \overrightarrow{H_r} \cos\theta_{i,r} = \overrightarrow{H_t} \cos\theta_t \quad \text{(VI.16)}$$

The magnetic field terms can be replaced with electric field terms according to the intrinsic impedance of the material (η) as follows:

$$E_i - E_r = E_t \frac{\eta_1 \cos\theta_t}{\eta_1 \cos\theta_{i,r}} \quad \text{(VI.17)}$$

The reflection coefficients « $\Gamma_\perp$ » and transmission coefficients « $\tau_\perp$ » are given successively by:

$$\boxed{\Gamma_\perp = \frac{E_r}{E_i} = \frac{\eta_2 \cos\theta_{i,r} - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_{i,r} + \eta_1 \cos\theta_t}} \quad \text{(VI.18)}$$

$$\boxed{\tau_\perp = \frac{E_t}{E_i} = 1 + \Gamma_\perp = \frac{2\eta_2 \cos\theta_{i,r}}{\eta_2 \cos\theta_{i,r} + \eta_1 \cos\theta_t}} \quad \text{(VI.19)}$$

VI.3.2. Parallel polarization

A plane wave has parallel polarization when the vector of the electric field of the incident, reflected, and transmitted waves is entirely localized in the plane of incidence (see Fig. VI.4) [14].

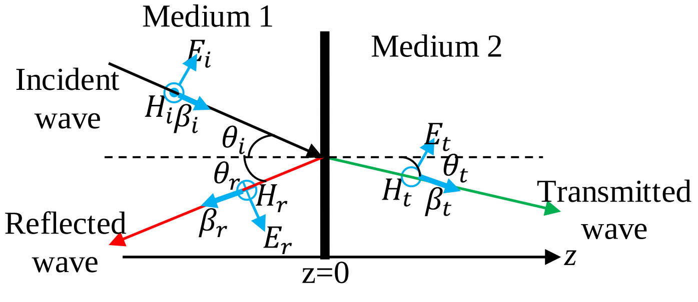


Fig.VI.4. Reflection and transmission of an obliquely incident wave on a flat interface with parallel polarization.

At z=0, the boundary conditions are written as:

$$\overrightarrow{E_{ix}} + \overrightarrow{E_{rx}} = \overrightarrow{E_{tx}} \quad \text{(VI.20)}$$

$$\overrightarrow{H_{iy}} - \overrightarrow{H_{ry}} = \overrightarrow{H_{ty}} \quad \text{(VI.21)}$$

Expressing both equations in terms of the total fields, we obtain:

$$H_i - H_r = H_t \quad \text{(VI.22)}$$

$$E_i \cos\theta_{i,r} + E_r \cos\theta_{i,r} = H_t \cos\theta_t \quad \text{(VI.23)}$$

And according to the intrinsic impedances, we obtain:

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2} \quad \text{(VI.24)}$$

The reflection and transmission coefficients in parallel polarization « $\Gamma_{//}$ » and «$\tau_{//}$ » are given successively by:

$$\boxed{\Gamma_{//} = \frac{E_r}{E_i} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_{i,r}}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_{i,r}}} \quad \text{(VI.25)}$$

$$\boxed{\tau_{//} = \frac{E_t}{E_i} = \frac{2\eta_2 \cos\theta_{i,r}}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_{i,r}}} \quad \text{(VI.26)}$$

VI.4. Critical angle

In the Snell's-Descartes's equation, total reflection occurs when : $\sin\theta_t = 1$. Therefore, from equation (VI.18): $\Gamma_{\perp} = 1$, and from equation (VI.25): $\Gamma_{//} = -1$.

« The critical angle θ_c» is the angle of incidence at which total reflection begins, such that:

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) \quad \text{(VI.27)}$$

From this equation, it is clear that to obtain a critical angle, it is necessary that : $n_1 > n_2$.

VI.5. Brewster's angle

« The Brewster's angle θ_B» is the angle of incidence at which total transmission occurs without any reflection.

In the condition : $\eta_1 \cos\theta_{i,r} = \eta_2 \cos\theta_t$, the reflection coefficient in parallel polarization $\Gamma_{//} = 0$.

Brewster's angle θ_B is given by:

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) \quad \text{(VI.28)}$$

General Conclusion

For a long time, electrical phenomena and magnetic phenomena were considered independent until the 1600s when William Gilbert articulated, in his work « De Magnete », the distinction between electric and magnetic bodies. The discovery in the 19th century by Ørsted, Ampère, and Faraday of the magnetic effects of electricity gradually led to the consideration that the 'electric' and 'magnetic' forces might actually be unified.

With the advent of the great scientist Maxwell, electromagnetism was born when, in 1865, he unified the predominant theories of electrostatics and magnetostatics into a single theory called the « Electromagnetic Field Theory ».

This theory of the electromagnetic field propelled science further and paved the way for significant discoveries. It laid the foundations for Albert Einstein's theory of special relativity in 1905. Moreover, the impact and benefits of Maxwell's electromagnetic field theory remain relevant to this day in various fields such as research in complex electromagnetic devices and installations, the field of communication, electromagnetic waves, radio waves, television, etc.

In this course, I have attempted to present to the student the foundations of electromagnetism and the basic principles and tribal requirements to understand electromagnetic phenomena. I have guided them progressively so that they can grasp the physical significance of the four Maxwell's equations and, consequently, reach the stage of mastering their application in the field of electromagnetic waves.

To achieve these objectives, this course has been divided into six chapters. The first chapter is dedicated to the study of electrostatic phenomena and important concepts such as electric field, Coulomb's law, Gauss's theorem, etc. The second chapter covers magnetostatics (magnetic field, Ampère's theorem, Lorentz force, Laplace force, etc.). Next, quasi-stationary regime is addressed in the third chapter, covering Faraday's law, Lenz's law, etc. Chapters 4, 5, and 6 are primarily intended for the study of the variable regime, including the integral and differential forms of Maxwell's equations, as well as the localized Ohm's law and the transition conditions of an electromagnetic field are organized in the fourth chapter. The study of the electromagnetic field propagation phenomenon, with mathematical and theoretical details, is the subject of the fifth chapter. The sixth and final chapter is dedicated to the reflection and transmission of electromagnetic waves in media. Additionally, the course is reinforced by a sufficient number of solved explanatory examples to further clarify the acquired concepts."

Finally, I welcome all remarks and constructive criticism from all students, teachers, and researchers in order to review and further improve this work.

ANNEX A: Useful relationships between operators

$$\overrightarrow{div}\left(\overrightarrow{curl}\vec{V}\right) = 0 \quad ;\forall\vec{V}$$

$$\overrightarrow{curl}\left(\overrightarrow{grad}f\right) = 0 \quad ;\forall f$$

$$\overrightarrow{div}\left(\lambda\vec{V}\right) = \lambda\overrightarrow{div}\vec{V} + \overrightarrow{grad}\lambda\vec{V}$$

$$\overrightarrow{curl}\left(\lambda\vec{V}\right) = \lambda\overrightarrow{curl}\vec{V} + \overrightarrow{grad}\lambda \wedge \vec{V}$$

$$\overrightarrow{curl}\left(\overrightarrow{curl}\vec{V}\right) = \overrightarrow{grad}\left(\overrightarrow{div}\vec{V}\right) - \vec{\Delta}\vec{V}$$

$$\overrightarrow{div}\left(\vec{A} \wedge \vec{B}\right) = \vec{B}\overrightarrow{curl}\vec{A} - \vec{A}\overrightarrow{curl}\vec{B}$$

$$\overrightarrow{div}\left(\vec{A} + \vec{B}\right) = \overrightarrow{div}\vec{A} + \overrightarrow{div}\vec{B}$$

$$\frac{\partial}{\partial t}\left(\overrightarrow{div}\vec{V}\right) = \overrightarrow{div}\frac{\partial\vec{V}}{\partial t}$$

ANNEX B: Lenz's Law.

Fig. 18-10a: The total flux Φ_1, surrounded by the coil, is directed as indicated, and it is increasing. The system seeks to oppose this growth. To achieve this, we imagine a current I_2, circulating in the coil to produce a flux Φ_2, opposing the growth of Φ_1. The flux Φ_2 must therefore have a direction opposite to that of Φ_1. Applying the right-hand rule (section 13.3), it follows that I_2 must circulate in the indicated direction. As the current flows toward terminal 2, it is (+) relative to terminal 1.

Fig. 18-10b: The total flux Φ_1 surrounded by the coil is directed in the same direction as in Fig. 18-10a, but it is decreasing. The system seeks to oppose this decrease. To achieve this, we imagine a current I_2 circulating in the coil to produce a flux Φ_2 opposing this decrease. The flux Φ_2 must therefore be oriented in the same direction as Φ_1. Applying the right-hand rule, it follows that I_2 must circulate in the indicated direction. As the current flows toward terminal 1, it is now (+) relative to terminal 2.

Fig. 18-10c: The total flux Φ_1 surrounded by the coil is oriented in the opposite direction to that of the two previous figures, and it is increasing. The system still seeks to oppose this growth. To achieve this, we imagine a current I_2 circulating in the coil to produce a flux Φ_2 opposing this growth and thus creating a flux Φ_2 in the opposite direction to that of Φ_1. Applying the right-hand rule, it follows that I_2 must circulate in the indicated direction. As the current flows toward terminal 1, it is (+) relative to terminal 2.

Fig. 18-10d: Finally, the total flux Φ_1 surrounded by the coil is directed in the same direction as in Fig. 18-10c, but it is decreasing. The system still seeks to oppose this decrease. To achieve this, we imagine a current I_2 circulating in the coil to produce a flux Φ_2 opposing this decrease and thus creating a flux in the same direction as Φ_1. Applying the right-hand rule, it follows that I_2 must circulate in the indicated direction. As the current flows toward terminal 2, it is (+) relative to terminal 1.

The same reasoning allows determining the polarity of the voltage appearing across the terminals of a winding or a coil in any circuit.

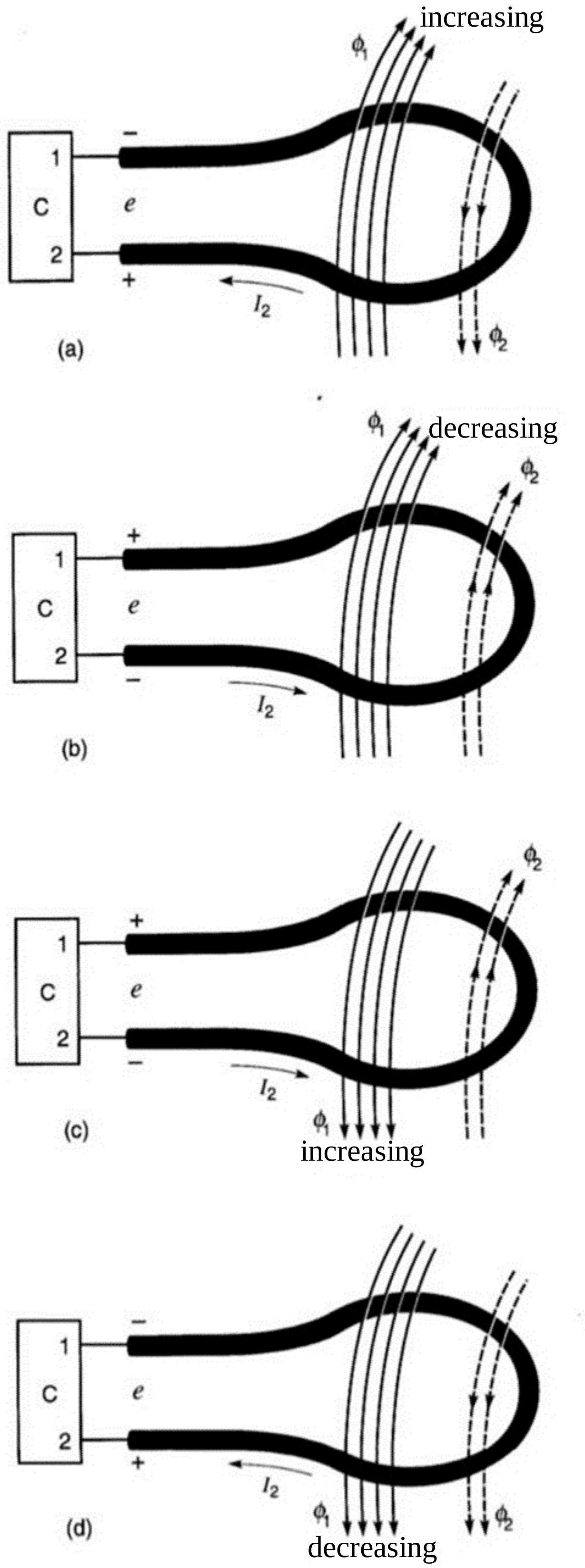
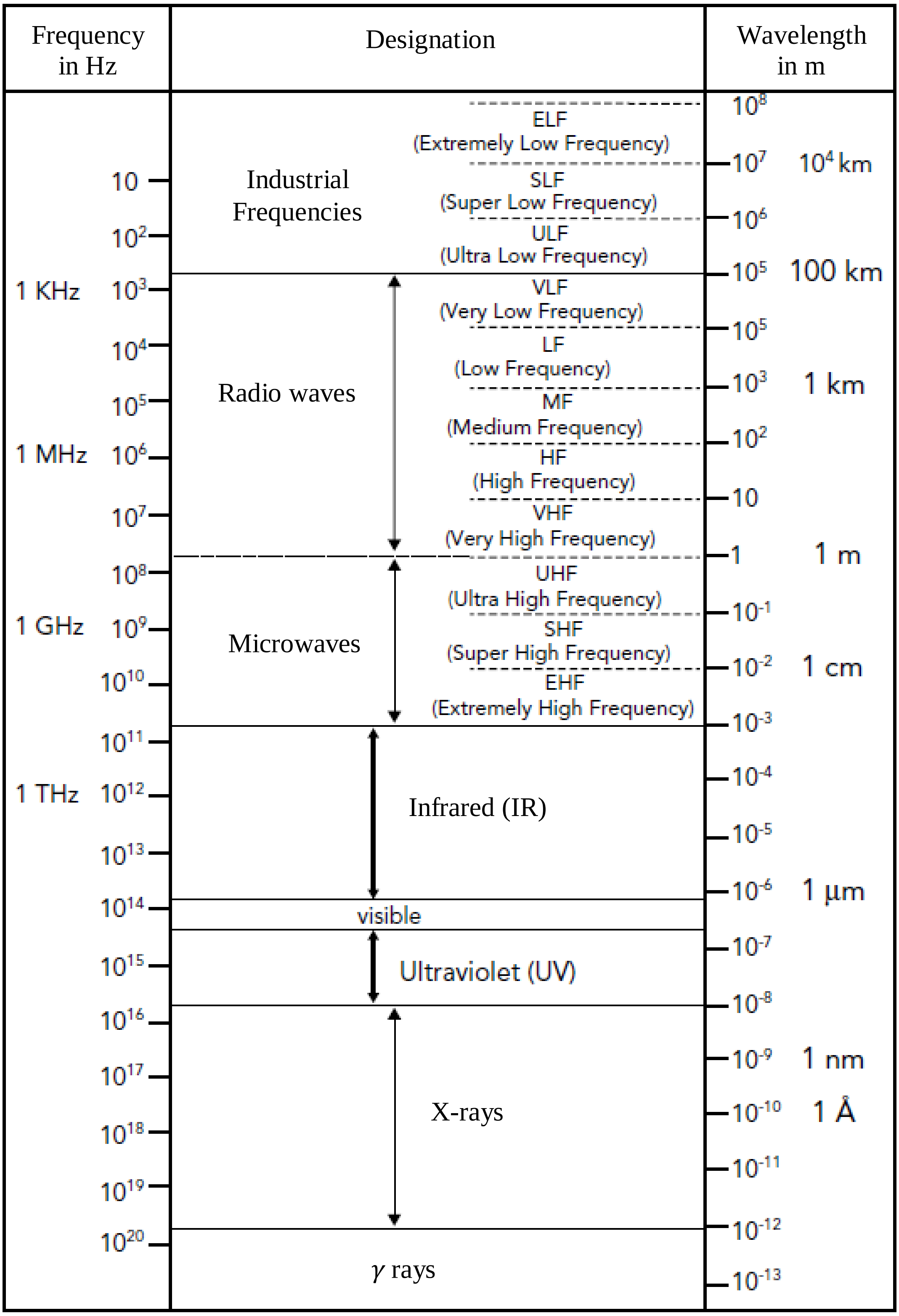
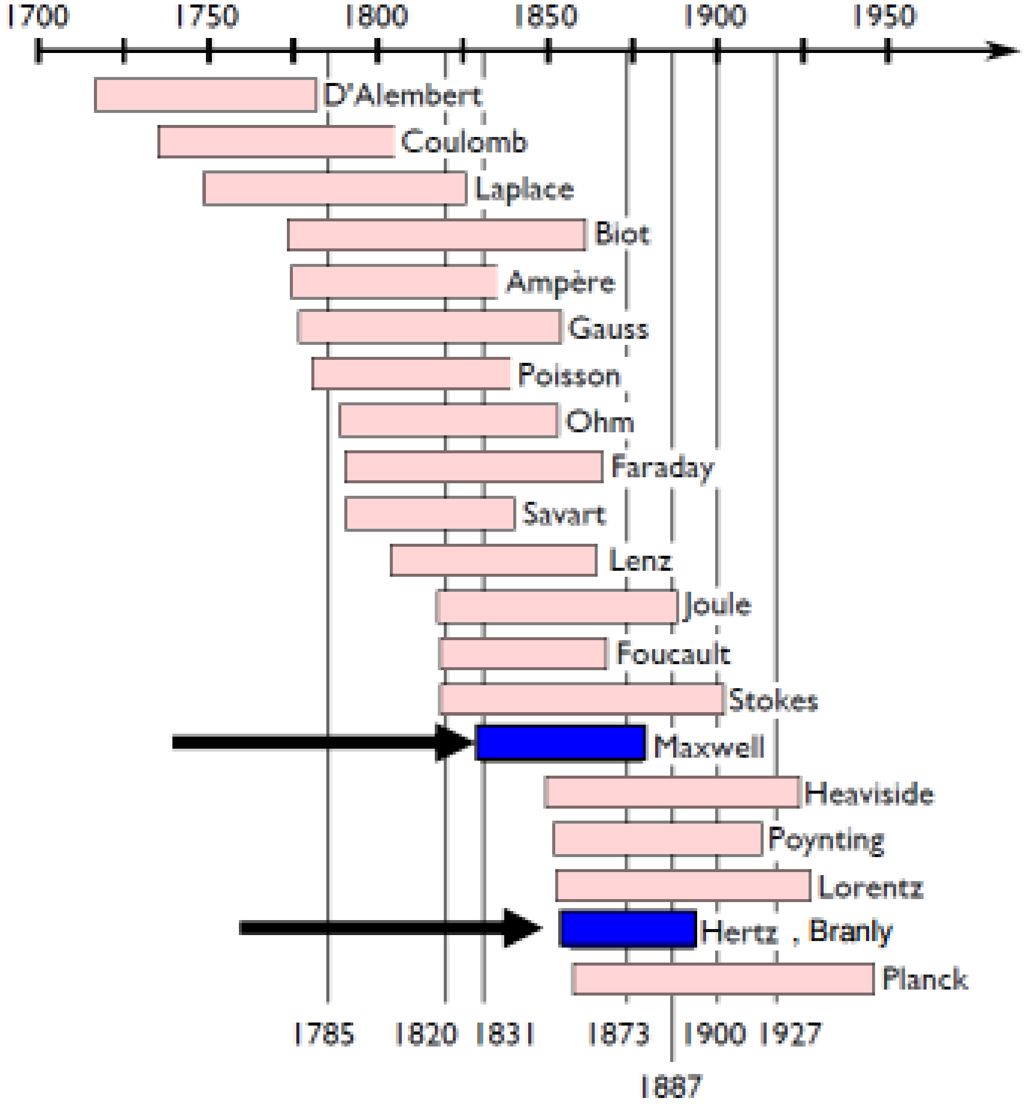


Figure 18-10. Diagrams illustrating the application of Lenz's Law to determine the polarity of an induced voltage.

ANNEX C: Electromagnetic Spectrum.



ANNEXE D: Chronology of scientists who contributed to the development of electromagnetism before 1950.



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